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RESEARCH ARTICLE

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Seismic stability analysis of a rock block using the block theory and Newmark method

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Summary

Seismic stability analysis of a rock block is an important issue in the field of rock mechanics and rock engineering. To solve this problem, the block theory and the Newmark method are combined, and a general method for seismic response of a rock block is introduced. First, the formation method of a three-dimensional rock block, which includes establishing a topological relationship among the block-polygon face-edge-vertex and dividing a complex block into convex subblocks, is presented, and the simplex integration method is employed to calculate the volume of a rock block and the areas of the polygonal faces. Second, the assumptions and algorithms of the general method for seismic response of a rock block are detailed. The dynamic analysis is carried out in time step. In each time step, the key technologies including analysis of the seismic force and motion mode, trial of the incremental displacement, check for the block entrance, and update of the motion parameters are performed in order. Last, two verification examples and three application examples including a wedge, a rock block of slope, and a combined rock block are used to analyze the correctness and practicality of the general method. The results show that, for a given ground motion record, the Newmark program can effectively simulate the dynamic response process of the motion mode, velocity, and safety factor of the rock block, and the permanent displacement under the earthquake action is obtained, which provides a quantitative parameter to evaluate the dynamic stability of a rock block.

KEYWORDS
block theory, Newmark method, rock block, safety factor and displacement, seismic dynamic analysis

1 | INTRODUCTION

Stability analysis of a rock block is an important topic in the field of rock mechanics and rock engineering,[1-4] the discontinuous numerical methods are usually used to solve this problem,[5-9] and the block theory is the most frequently choice. Block theory was early proposed by Dr Shi in the 1970s,[10,11] later in 1985, professor Goodman and Dr Shi wrote their first book, which heralded the formal formation of the block theory.[12] Since it was proposed, the block theory had rapidly developed in such areas as block identification, uncertainty analysis, and support design,[13-17] and was also wildly used in many fields, including the slopes, tunnels, and dam foundations.[18-24] The existing researches were mainly static analysis, and the earthquake action is reduced to a constant inertia force imposed on the rock block, which is called as the quasi-static method.[25] This approach is simple in calculation and only relates to the potential earthquake intensity in the region where the project is located. However, determining the seismic inertia force requires extensive engineering experience, and the quasi-static method can not calculate the dynamic response process of the rock block under the earthquake action.

The Newmark method, proposed by Newmark in 1965,[26] assumes that the sliding block yield a permanent displacement when the ground acceleration exceeds a critical value, and it is initially used for seismic dynamic response of the dams and embankments.[27,28] In the Newmark method, a dam or an embankment is treated as a rigid sliding block, the earthquake action is considered as a dynamic inertial force that changes with time, and several small time step calculations are implemented to realize the whole process analysis of the seismic response of the rigid sliding block; in a time step, the acceleration is obtained by solving the mechanical equilibrium equations, and the displacement is obtained by double integration. This method has a clear concept and a simple calculation process, and the permanent displacement can be used as a quantitative parameter for analyzing the dynamic stability of the dams and embankments, which breaks the traditional concept of assessing dynamic stability using the safety factor.

In fact, the Newmark method treats the dams and embankments as rigid sliding blocks, which is as the same approach as the block theory. Thus, the Newmark method is also suitable for analyzing the seismic dynamic response of a rock block. Hendron[29] early studied the movement of the rock slope under dynamic loading using the Newmark method. On the basis of laboratory tests, Wartman et al[30,31] proved that the Newmark method could give reasonable predictions. Wilson and Keefer[32] applied the Newmark method to predict the displacement of a landslide triggered by the 1979 Coyote Creek, California earthquake, and the result agreed well with the observed displacement, which also proved that the Newmark method is effective for analyzing the seismic dynamic response of the nature slope.

At present, some scholars had already done some work on applying the Newmark method to analyze the seismic dynamic response of a rock block. Chang and Lin et al[33,34] calculated the displacements of blocks and slopes subjected to strong motions. Combining the Newmark method with the conventional limit equilibrium analysis, Ghosh and Haupt[35] developed a computer program to compute the displacement of a rock wedge induced by earthquake. Ling and Leshchinsky[36] extended a rotational limit equilibrium method for determining the permanent displacement of a slope under seismic excitation, and the rational criterion based on permanent displacement is proposed to evaluate the seismic performance of the slope. To analyze the dynamic response of a sliding wedge subject to random seismic excitations, Ling and Cheng[37] extended the Hoek-Bray procedure to seismic conditions. Wang and Lin[38] developed the regional landslide potential mapping procedure with Newmark displacement, and the procedure is verified by the case histories of slope failures caused by the 1999 Chi-Chi earthquake. Jibson[39] systemically summarized the applications of the Newmark method, and a quantitative criteria of the permanent displacement for evaluation of the seismic dynamic stability of a slope was investigated. Through a number of Newmark displacement calculations using the ground motion records, Roy et al[40] discussed the influence of strong motion characteristics on the permanent displacement of a slope.

These existing studies indicate that the Newmark method is useful for the characteristics of a rock block under earthquake action. However, most of the studies including the two-dimensional analysis of a slope and three-dimensional analysis of a wedge, are based on the limit equilibrium analysis of simple sliding face, which can not be used for a complex rock block. To solve this problem, taking the inherent advantage of combination of the block theory and the Newmark method, a general method for analyzing the seismic dynamic response of a rock block is introduced. In this paper, the theories of this general method, including the geometric operation of a three-dimensional rock block, the calculation process, and the key technologies of the algorithm, are detailed.

2 | FORMING A THREE-DIMENSIONAL ROCK BLOCK

2.1 | Description of a three-dimensional rock block

Usually, a rock block is formed by cutting the rock with several structural planes and free planes. A structural plane or free plane can be expressed by dip angle α, dip direction β, and the three-dimensional coordinates of a point in the plane (x_0, y_0, z_0), and the equation of the plane can be written as

$$\sin\alpha\sin\beta\cdot x + \sin\alpha\cos\beta\cdot y + \cos\alpha\cdot z = D, \tag{1}$$

where $(\sin\alpha\sin\beta, \sin\alpha\cos\beta, \cos\alpha)$ is the unit normal vector of the plane; D represents location of the plane and can be determined with (x_0, y_0, z_0).

A structural plane or free plane divides the space into two parts including the upper and lower half spaces,[12] and the equations of the parts are

$$\sin\alpha\sin\beta\cdot x+\sin\alpha\cos\beta\cdot y+\cos\alpha\cdot z\geq D\quad(\text{Upper half space}),\tag{2}$$

$$\sin\alpha\sin\beta\cdot x+\sin\alpha\cos\beta\cdot y+\cos\alpha\cdot z\leq D\quad(\text{Lower half space}).\tag{3}$$

A rock block can be denoted by the digital coding method,[12] which records the upper or lower half spaces of the structural planes and free planes. In this method, 0 denotes the upper half space of the plane, 1 denotes the lower half space of the plane, and 2 denotes that the plane is not a member of the rock block. Theoretically, the digital coding method is based on the assumption of infinite plane, and the digital coding method can just be used for a convex block. Although the basic assumption of the block theory is that the structural plane is infinite, the free plane may be a finite plane, which means that, the rock block may be concave. For the geometric description of such a complex rock block, the combination method is employed.[41-43] In the combination method, the complex rock block is divided into several convex subblocks, and these convex subblocks are connected with several common planes. In Figure 1, the complex rock block is formed by cutting the rock with 16 planes marked with A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, and P, where E, I, N, and P are the structural planes and the others are free planes. The complex rock block can be divided into three convex subblocks. The subblock 1 is formed by cutting the rock with six planes including A, B, C, D, E, and F, and the digital coding is 1001002222222222; the subblock 2 is formed by cutting the rock with six planes including F, G, H, I, J, and K, and the digital coding is 2222210001022222; the subblock 3 is formed by cutting the rock with six planes including K, L, M, N, O, and P, and the digital coding is 2222222222100000.

2.2 | Geometric representation of the polygonal faces

Once the equation of each structure or free plane and the digital coding are determined, the polygonal faces consisting a rock block can be analyzed using the computational geometry theory.[44] It is assumed that the number of the structural and free planes forming the rock block is n. First, we need to calculate all the vertices of the rock block. Each vertex is formed by the intersection among three planes, and the number of the possible vertices is C_n^3. For any three planes, i, j, and l, as shown in Figure 2, the intersection can be obtained by solving the following equations:

$$\begin{cases}\sin\alpha_i\sin\beta_i\cdot x+\sin\alpha_i\cos\beta_i\cdot y+\cos\alpha_i\cdot z=D_i\\ \sin\alpha_j\sin\beta_j\cdot x+\sin\alpha_j\cos\beta_j\cdot y+\cos\alpha_j\cdot z=D_j\\ \sin\alpha_l\sin\beta_l\cdot x+\sin\alpha_l\cos\beta_l\cdot y+\cos\alpha_l\cdot z=D_l\end{cases}.\tag{4}$$

Then, for each structural or free plane of the rock block, the attributes of all vertices in the plane, including the coordinates and the number of the intersecting planes, should be recorded. Last, the vertices in each plane can be arranged

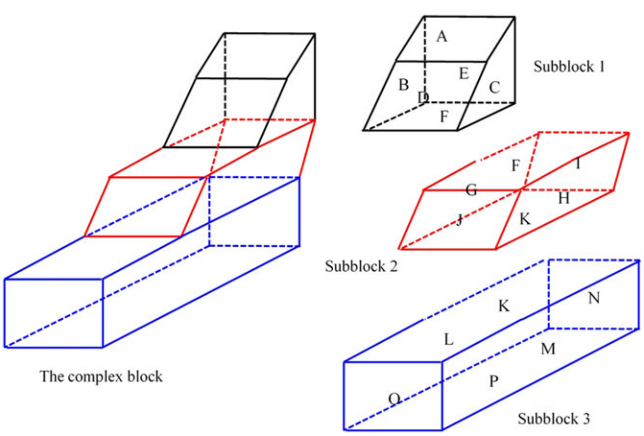

FIGURE 1 The combination method for describing a complex rock block with several convex subblocks [Colour figure can be viewed at wileyonlinelibrary.com]

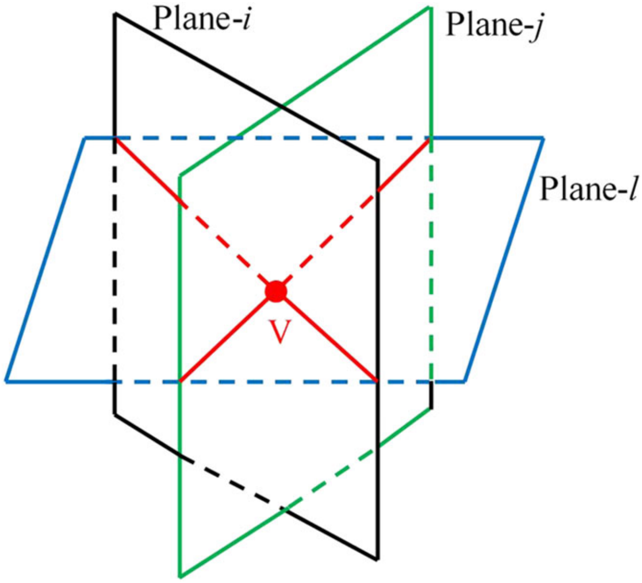

FIGURE 2 A vertex of the rock block by calculating the intersection among three planes [Colour figure can be viewed at wileyonlinelibrary.com]

in order to identify the edges, and all the polygonal faces consisting the rock block are expressed explicitly. Specifically, for a given structural or free plane, starting from an arbitrary vertex in it, the next vertex is found with a rule, which is that the two vertices of an edge have two common intersecting planes.

2.3 | Calculations of a rock block with simplex integration method

The rock block is composed of several polygonal faces, each polygonal face is composed of several edges, each edge is composed of two vertices, and each vertex is determined by the three-dimensional coordinates; the vertices in each polygonal face are stored in a counterclockwise order. Using the explicit expression of the rock block, the analytical calculation of the volume of the rock block and the area of each polygonal face can be implemented with the simplex integration method.[45]

For a polygonal face, it consists of a number of two-dimensional simplexes, and the area of the polygon face is equal to the sum of the areas of these two-dimensional simplexes. An arbitrary two-dimensional simplex is a directed triangle recorded as $P_0P_1P_2$, and its area can be calculated as

$$J = \frac{1}{2!}\left(\overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2}\right)\cdot \overrightarrow{n}_i, \tag{5}$$

where $\overrightarrow{n}_i$ is the unit normal vector of the polygonal face, $\overrightarrow{P_0P_i} = (x_i - x_0, y_i - y_0, z_i - z_0)$, and (x_k, y_k, z_k) is the three-dimensional coordinates of the vertex P_i, k = 0,1,2, …. The area of the polygon face is written as

$$S = \sum_k \frac{1}{2!}\left(\overrightarrow{P_0P_k} \times \overrightarrow{P_0P_{k+1}}\right)\cdot \overrightarrow{n}_i, \tag{6}$$

where *k is* the number of two-dimensional simplexes consisting the polygonal face.

For a rock block, it consists of a number of three-dimensional simplexes, and the volume of the rock block is equal to the sum of the volumes of these three-dimensional simplexes. An arbitrary three-dimensional simplex is a directed tetrahedron recorded as $P_0P_1P_2P_3$, and its volume can be calculated as

$$J = \frac{1}{3!}\begin{vmatrix} 1 & x_0 & y_0 & z_0 \\ 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \end{vmatrix}. \tag{7}$$

If P_0 is taken as the origin with the coordinates (0, 0, 0), the volumetric operation of the tetrahedron can be reduced to a two-dimensional form. It means that the volume of the tetrahedron can be calculated using its surface triangles. Then, the volume of the rock block can be calculated as

$$V = \sum_{p}\sum_{k_p}\frac{1}{3!}\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}, \tag{8}$$

where k_p *is* the number of directed triangles consisting a polygonal face and p is the number of the polygonal faces consisting the rock block.

3 | ASSUMPTIONS AND ALGORITHMS OF THE NEWMARK METHOD FOR A ROCK BLOCK

To analysze the seismic dynamic response of a rock block using the Newmark method, it should abide by the assumptions of the block theory and the Newmark method simultaneously,[12,26] and the assumptions mainly include the following contents:

1. The rock block is treated as a rigid body, which means that the block deformation and distortion will not be considered.
2. The structural planes that are cutting the rock block are perfectly planar and infinite, which guarantee that block morphology can be described by linear vector equations and no discontinues will terminate within the region of a rock block.
3. Only the studied rock block may move, the motion mode is translation, and the surrounding rock mass is fixed.
4. The dynamic response analysis is divided into many small time steps, and in each time step, the displacement of the rock block is small.
5. The static and dynamic shearing resistances of the sliding faces are taken to be the same, which means that the shear strength parameters, including the cohesion force and friction angle, are constant.

Combining of the block theory and the Newmark method, Dr Shi has early-written a program named NM, and provided the technical manual to let the users know how to use this program.[46,47] However, the theory of the program has not been reported so far. According to Dr Shi's technical manual, we rewrite the NM program with C language. The analysis process of NM is shown in Figure 3, and the program mainly includes the following steps:

1. Input the geometric and mechanical parameters of the structural planes, the volume of the rock block, the density of the rock, and the gravity acceleration.
2. Analyze the vectors of edges of the joint piercer, and compute all possible sliding directions of the single face sliding and double faces sliding.
3. Input the acceleration data of a ground motion record, and divide them into small time steps with equal interval.
4. Initialize the motion mode and parameters such as displacement and velocity.
5. Begin a new time step analysis, and analyze the possible motion mode and incremental displacement based on the forces acting on the rock block.
6. Calculate the position of the rock block with the possible incremental displacement, check for the block entrance considering boundary constraint conditions, and the real incremental displacement and velocity of the rock block are determined.
7. Compute the acceleration and the safety factor of the rock block, if the time step does not run out, go to 5.
8. End of the seismic dynamic analysis of the rock block.

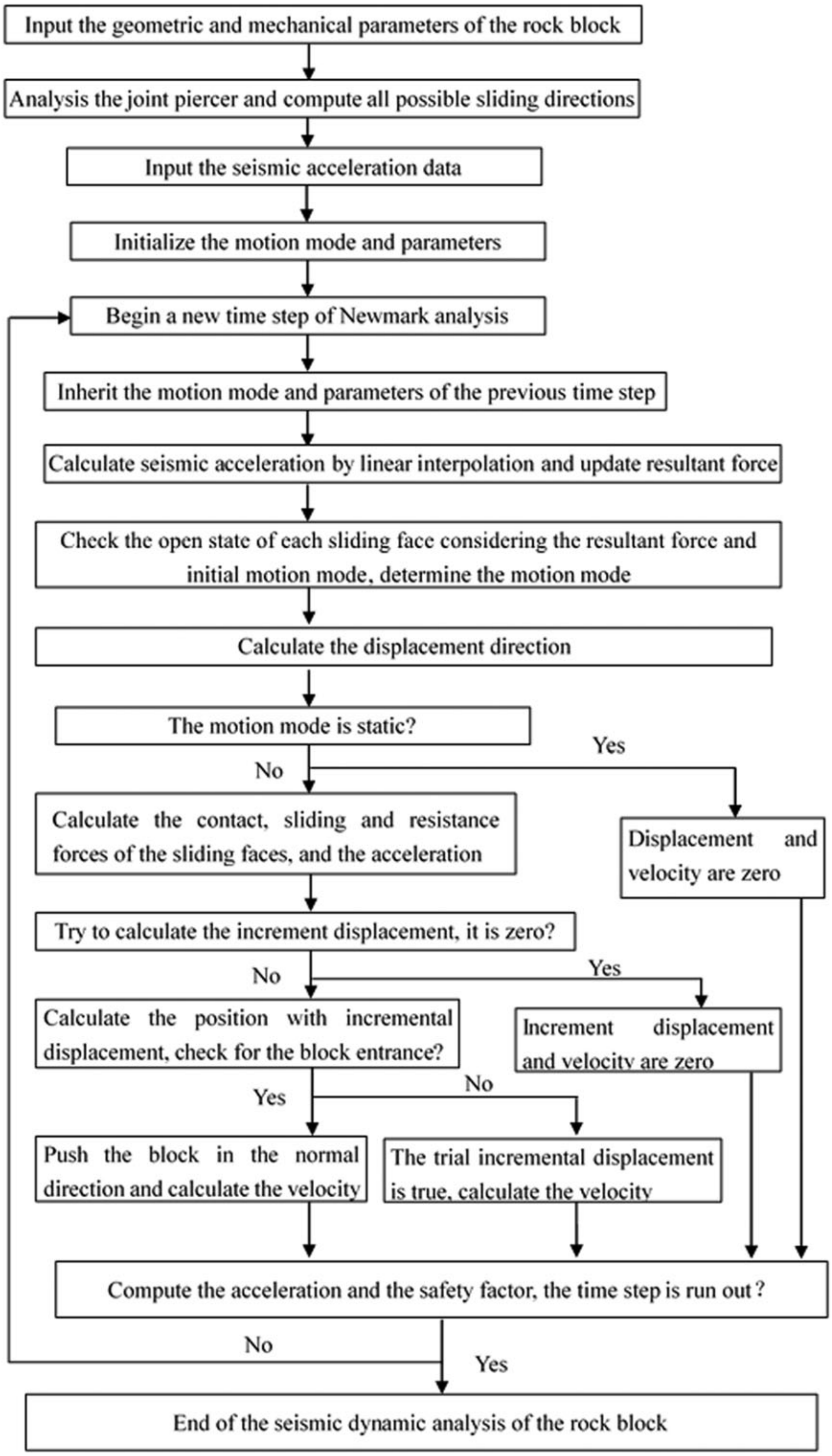

FIGURE 3 Seismic dynamic analysis process of a rock block with the Newmark program

4 | KEY TECHNOLOGIES OF THE NEWMARK METHOD FOR A ROCK BLOCK

4.1 | Analysis of the seismic force and motion mode

In the Newmark method, the earthquake action is considered as the inertial force, which is a dynamic loading that changes with time and needs to be updated in each time step during the dynamic analysis.

In general, the acceleration data of the ground motion is measured at a certain time interval in the local coordinate system, where the axes X and Y are horizontal and the axis Z is vertical. Before the seismic dynamic analysis, the data measured in the local coordinate system must be converted to the standard coordinate system, where the axis X is horizontal and points to the east, Y is horizontal and points to the north, and Z is vertical and points upward. The coordinate conversion method is as follows:

$$\begin{Bmatrix} a_x \\ a_y \\ a_z \end{Bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta & 0 \\ \cos\delta & \sin\delta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} a_X \\ a_Y \\ a_Z \end{Bmatrix}, \tag{9}$$

where δ is the angle from the north clockwise to the X axis, a_x, a_y, a_z, a_X, a_Y, and a_Z are the components of seismic acceleration in each axis of the standard and local coordinate systems, respectively.

Let the time step of the Newmark method as Δt, and the inertial force acting on the rock block in the time step n, is:

$$f_s = m\begin{Bmatrix} a_{xn} \\ a_{yn} \\ a_{zn} \end{Bmatrix} = m\frac{t_2 - n\Delta t}{t_2 - t_1}\begin{Bmatrix} a_{x1} \\ a_{y1} \\ a_{z1} \end{Bmatrix} + m\frac{n\Delta t - t_1}{t_2 - t_1}\begin{Bmatrix} a_{x2} \\ a_{y2} \\ a_{z2} \end{Bmatrix}, \tag{10}$$

where m is the mass of the rock block, a_{xn}, a_{yn}, and a_{zn} are the seismic acceleration components in the time step n, t_1 and t_2 are the two closest measured times of the ground motion with $n\Delta t$, and a_{x1}, a_{y1}, a_{z1}, a_{x2}, a_{y2} and a_{z2} are the seismic acceleration components at time t_1 and t_2, respectively.

In each time step of the Newmark method, the equilibrium equations for the forces acting on the rock block can be established as

$$M\cdot\vec{s} = \vec{r} + \sum_i N_i\vec{n_i} - T\vec{s},$$

$$T = \sum_i (N_i \tan\varphi_i + c_i A_i),$$

$$S\cdot\vec{s} = \vec{r} + \sum_i N_i\vec{n_i}, \tag{11}$$

where M is the driving force; $\vec{s}$ is sliding direction; $\vec{r}$ is resultant force; N_i is the normal force acting on the sliding surface i; $\vec{n_i}$ is the unit normal vector of the sliding surface i pointing to the inside; T is the resistant force; S is the sliding force; and φ_i, c_i, and A_i are the friction angle, cohesion force, and area of the sliding surface i, respectively.

According to the block theory, there may be three kinds of motion modes of a rock block: free falling, single face sliding, and double faces sliding, and the calculation method of the sliding direction, sliding force, resistant force, and the normal force are detailed by Goodman and Shi.[12] Different from the traditional block theory and the Newmark method, the motion mode of the rock block may be changed in each time step. At the beginning of a time step, the motion mode of the previous time step is inherited as the initial motion mode. If the initial motion mode is free falling, the motion mode does not change in the current time step. If the initial motion mode is single face sliding or double faces sliding, the normal force acting on the sliding surface is checked to determine whether the sliding surface is open, and the motion mode in the current time step may be free falling, single face sliding, or double faces sliding. Apart from these conditions, when the total displacement of the rock block may be zero, the initial motion mode is defined as static. In this condition,

1. If $\vec{r} = 0$, the motion mode is still static.
2. If all structural planes satisfy the following condition, the motion mode is free falling.

$$\vec{r}\cdot\vec{n_i} > 0. \tag{12}$$

3. The motion mode is sliding, and all the possible sliding directions of the single face sliding and double faces sliding are calculated. The most unfavorable sliding direction is chosen with the following condition:

$$\vec{s} = \max\left(\vec{r}\cdot\vec{s_i}, \vec{r}\cdot\vec{s_{ij}}\right), \tag{13}$$

where $\vec{s_i}$ and $\vec{s_{ij}}$ are the sliding directions of a single face sliding and a double faces sliding, respectively.

4.2 | Newmark displacement

4.2.1 | Trial of the incremental displacement

Since the possible sliding direction is determined, the acceleration of the rock block in the current time step can be obtained as

$$\vec{a} = \frac{S}{m}\vec{s}. \tag{14}$$

The motion direction of the rock block is

$$\frac{\vec{e} = \vec{v}_0\Delta t + \vec{a}\Delta t^2/2}{\left|\vec{v}_0\Delta t + \vec{a}\Delta t^2/2\right|}, \tag{15}$$

where $\vec{e}$ is the motion direction, $\vec{v}_0$ is the initial velocity, and $\vec{a}$ is the acceleration.

Considering the friction and cohesion of the sliding face, the incremental displacement of the rock block in the current time step can be obtained as

$$\Delta\vec{d} = \vec{v}_0\Delta t + \vec{a}\Delta t^2 - \frac{T\Delta t^2}{2m}\vec{e}, \tag{16}$$

where $\Delta\vec{d}$ is the incremental displacement. If $\Delta\vec{d}\cdot\vec{e} < 0$, the rock block stops, otherwise the rock block moves.

4.2.2 | Check for the block entrance

With reference to the position of the rock block before the Newmark analysis, the total displacement at the end of the current time step $n + 1$ is

$$\vec{d}_{n+1} = \vec{d}_n + \Delta\vec{d}_{n+1}, \tag{17}$$

where $\vec{d}_n$ and $\vec{d}_{n+1}$ are the total displacements at the end of time step n and time step $n + 1$, respectively. $\Delta\vec{d}_{n+1}$ is the incremental displacement in the time step $n + 1$.

Once the rock block moves in the current time step, the block entrance should be checked with the position constraint of the rock block at the end of the current time step. If the total displacement at the end of the time step $n + 1$ satisfies the inequality (18), it means that the rock block moves away from the initial position, and the position constraint is automatically satisfied.

$$\vec{d}_{n+1}\cdot\vec{n}_i \geq 0. \tag{18}$$

Otherwise, the block entrance happens, and the entered displacement needs to be pushed out in the nearest direction, which means that the displacement component in the sliding direction is preserved and the displacement component in the normal direction is ignored. In this case, all the possible sliding directions of the single face and double faces sliding need to be calculated, and the most unfavorable motion direction is chosen by Equation 19.

$$\vec{s}_{n+1} = \max\left(\vec{e}_{n+1}\cdot\vec{s_i}, \vec{e}_{n+1}\cdot\vec{s_{ij}}\right), \tag{19}$$

where $\vec{s}_{n+1}$ is the real sliding direction in the time step $n + 1$.

4.2.3 | Update of the motion parameters

If the motion mode of the rock block in the current time step is static, the motion parameters are

$$\begin{aligned}\vec{d}_{n+1} &= 0,\\ \vec{v}_{n+1} &= 0,\end{aligned} \tag{20}$$

where $\vec{v}_{n+1}$ is the velocity at the end of time step $n + 1$.

If the rock block is stopped by the friction and cohesion in the current time step, the motion parameters are

$$\begin{aligned} \vec{d}_{n+1} &= \vec{d}_n, \\ \vec{v}_{n+1} &= 0. \end{aligned} \tag{21}$$

If the rock block moves (free falling or sliding) in the current time step and satisfies the boundary constraint conditions, the motion parameters are

$$\begin{aligned} \vec{d}_{n+1} &= \vec{d}_n + \Delta \vec{d}_{n+1}, \\ \vec{v}_{n+1} &= \frac{2\Delta \vec{d}_{n+1}}{\Delta t} - \vec{v}_n, \end{aligned} \tag{21}$$

where $\vec{v}_n$ is the velocity at the end of time step n.

If the rock block moves (free falling or slide) in the current time step, and the block entrance exists, the motion parameters are

$$\begin{aligned} \vec{d}_{n+1} &= \vec{s}_{n+1} \cdot \left(\vec{d}_n + \Delta \vec{d}_{n+1}\right) \cdot \vec{s}_{n+1}, \\ \vec{v}_{n+1} &= \vec{s}_{n+1} \cdot \left(\frac{2\Delta \vec{d}_{n+1}}{\Delta t} - \vec{v}_n\right) \cdot \vec{s}_{n+1}. \end{aligned} \tag{22}$$

Once the motion parameters including the displacement and the velocity are determined, the acceleration and safety factor of the rock block in the current time step can be calculated as

$$\begin{aligned} \vec{a}_{n+1} &= \frac{\vec{v}_{n+1} - \vec{v}_n}{\Delta t}, \\ F_{n+1} &= \frac{T}{S + m\left|\vec{a}_{n+1}\right|}, \end{aligned} \tag{23}$$

where $\vec{a}_{n+1}$ and F_{n+1} are the acceleration and the safety factor, respectively, in the time step $n + 1$.

5 | EXAMPLES

5.1 | Verification examples

Verification example 1 and verification example 2 are used to check the correctness of the Newmark method. The geometric and mechanical parameters of the structural and free planes for the verification examples are shown in Table 1. The density of the rock is 2400 kg/m^3, and the gravity acceleration is 9.8 m/s^2. The volumes of the rock blocks of verification example 1 and verification example 2 are 216.51 m^3 and 62.50 m^3, respectively, and Figure 4 shows the geometric shape of the rock blocks.

TABLE 1 The geometric and mechanical parameters of the rock block of the verification examples

	Verification Example 1				Verification Example 2				
Structural and free planes	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**	**4**
α (°)	45	45	45	0	45	90	90	0	90
β (°)	0	120	240	0	270	90	0	270	180
A point x_0,y_0,z_0 (m)	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, −5	0, 0, 0	0, 0, 0	0, 0, 0	0, 5, 5	0, 5, 5
Friction angle (°)	30	30	30	-	45/30	-	-	-	-
Cohesion force (Kpa)	0	0	0	-	0	-	-	-	-
Area (m^2)	61.24	61.24	61.24	129.90	35.36	25.00	12.50	25.00	12.50

In the verification example 1, the block is formed by cutting the rock with three structural planes 0, 1, 2 and one free plane 3. The digital coding is 1110, and no earthquake loads are applied. Theoretically, the motion mode of the rock block is free falling, and the displacement is $0.5*9.8*t_0^2$ m, where t_0 is the time of block movement. The NM program

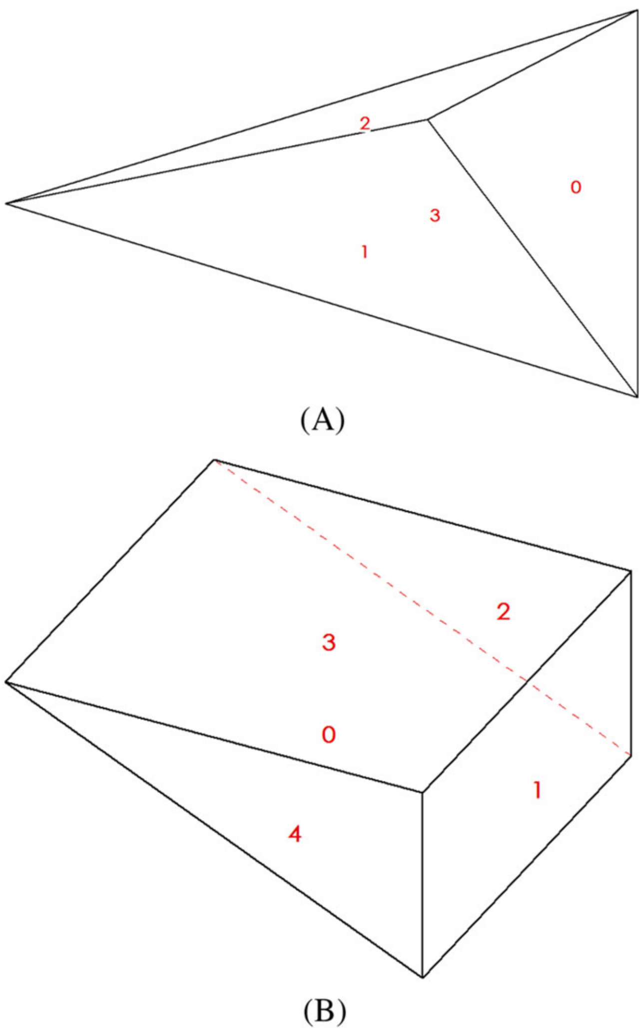

FIGURE 4 Geometric shape of the rock blocks A, verification example 1; B, verification example 2 [Colour figure can be viewed at wileyonlinelibrary.com]

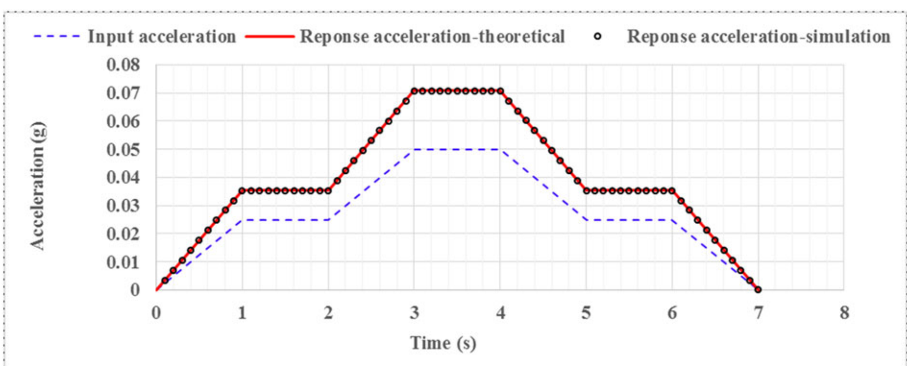

FIGURE 5 The input seismic acceleration and the response acceleration of the rock block [Colour figure can be viewed at wileyonlinelibrary.com]

runs for 1 second, and the permanent displacement of the rock block is 4.90 m, which is consistent with the theoretical solution.

In verification example 2, the block is formed by cutting the rock with one structural planes 0 and four free planes 1, 2, 3, and 4. The digital coding is 00010, and the motion mode of the rock block is single face sliding. For case 1, the friction angle of the sliding face is 45°, and no earthquake loads are applied. Theoretically, the safety factor is 1 and the rock block is static. For case 2, the friction angle of the sliding face is 30°, and no earthquake loads are applied. Theoretically, the safety factor is 0.406 and the displacement is $0.5*0.30*9.8*t_0^2$ m. For case 3, the friction angle of the sliding face is 45°, and the earthquake loads in the X direction are applied. It is assumed that the seismic acceleration is a_0*g (gravity acceleration) at t_0, theoretically, the sliding force, the resistant force, and the acceleration of the rock block can be calculated by Equation 24.

TABLE 2 The geometric and mechanical parameters of the structural and free planes for the wedge

Structural and Free Planes	α (°)	β (°)	A Point x_0, y_0, z_0 (m)	Friction Angle (°)	Cohesion Force (Kpa)	Area (m^2)
0	35	110	0, 0, 0	30	0	96.10
1	30	225	0, 0, 0	30	0	146.51
2	60	180	0, 0, 0	-	-	64.05
3	5	180	1.5, 4.5, 6	-	-	174.24

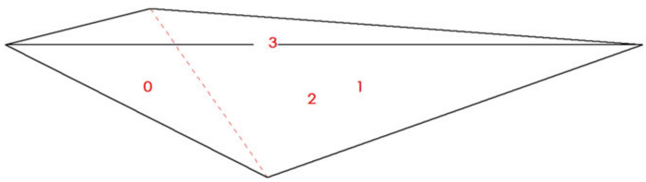

FIGURE 6 Geometric shape of the wedge [Colour figure can be viewed at wileyonlinelibrary.com]

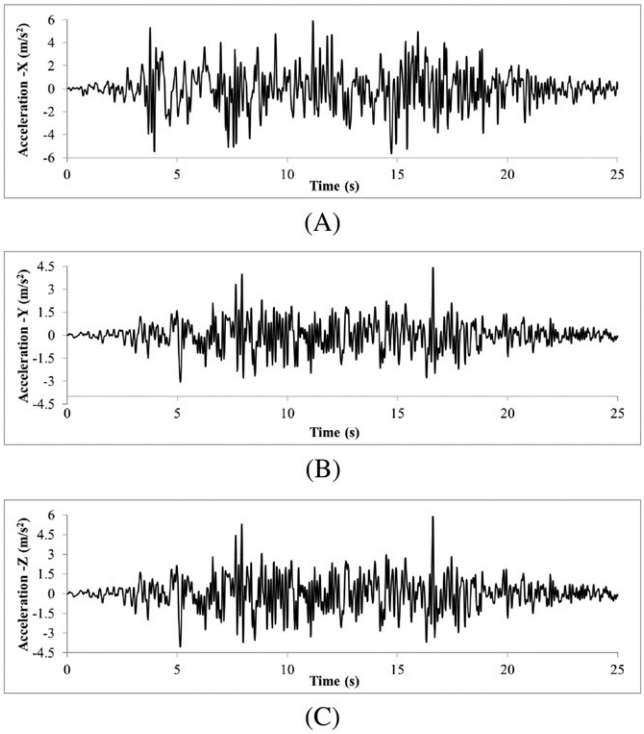

FIGURE 7 Acceleration components of the input ground motion A, X direction; B, Y direction; and C, Z direction

$$S = mg\sin\alpha + ma_0 g\cos\alpha,$$
$$T = (mg\cos\alpha - ma_0 g\sin\alpha)\tan\varphi, \quad (24)$$
$$a = \frac{S-T}{m} = (\sin\alpha + a_0\cos\alpha - \cos\alpha + a_0\sin\alpha\tan\varphi)g = \sqrt{2}a_0 g.$$

Figure 5 shows the input seismic acceleration in the X direction and the theoretical solution of response acceleration of the rock block. For the three cases, the simulation results of the NM program are consistent with the theoretical solution, which has proved the correctness of the Newmark method.

5.2 | A wedge

In this example, a movable wedge is formed by cutting the rock with two structural planes 0 and 1 and two free planes 2 and 3. The geometric and mechanical parameters of the structural and free planes are shown in Table 2. The density of the rock is 2400 kg/m^3, and the gravity acceleration is 9.8 m/s^2. The digital coding is 0011, Figure 6 shows the wedge, and the coordinates of the four vertices are (0, 0, 0), (−3.46, 21.89, 7.52), (−7.73, 3.41, 5.90), and (11.05, 3.41, 5.90), respectively. The areas of the four polygonal faces are 96.10 m^2, 146.51 m^2, 64.05 m^2, and 174.24 m^2, respectively. The volume of the wedge is 324.38 m^3. The motion mode of the wedge is double faces sliding due to its own weight, and $\vec{s} = (0.148, 0.935, -0.321)$, $S = 2.452$MN, $T = 4.674$MN, $N_1 = 3.613$MN, $N_2 = 4.482$MN, and $F = 1.906$.

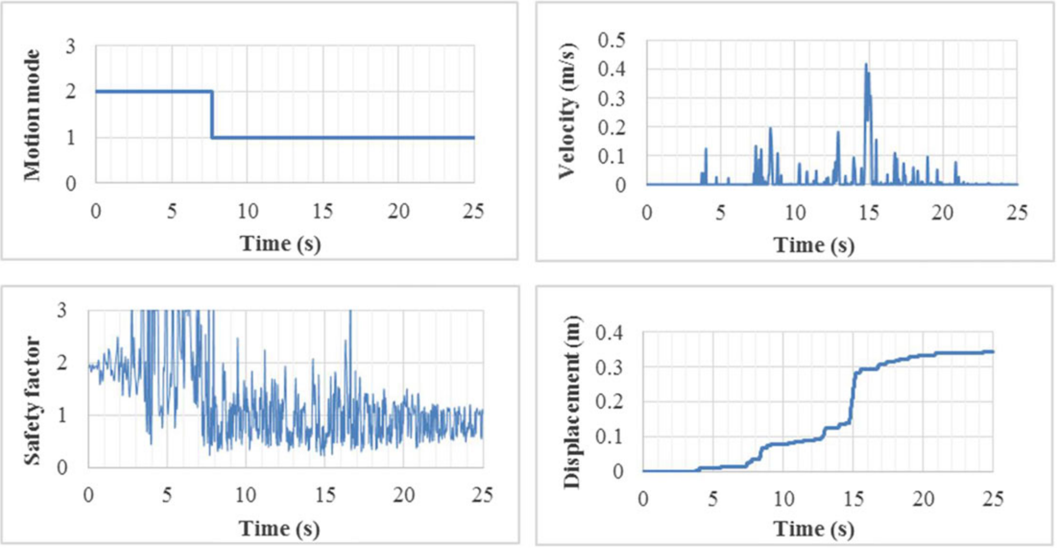

FIGURE 8 Seismic dynamic response of the wedge [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 3 The geometric and mechanical parameters of the structural and free planes for the slope block

Structural and Free Planes	α (°)	β (°)	A Point x_0, y_0, z_0 (m)	Friction Angle (°)	Cohesion Force (Kpa)	Area (m^2)
0	90	180	0, 0, 0	30	0	12.53
1	75	135	0, 0, 0	35	0	14.39
2	60	110	−1.25, −1.25, −1.0	35	0	6.16
3	75	225	2.5, 0, 0	33	0	14.39
4	60	250	3.75, −1.25, −1.0	33	0	6.16
5	25	180	0, 0, 0	30	0	17.46
6	18	180	−1.25, −1.25, −0.875	28	0	6.61
7	5	180	0, 0, 3.33	-	0	24.26
8	75	180	−1.0, −4.125, −1.0	-	0	44.67

Figure 7 shows the acceleration components of the input ground motion in the X, Y, and Z directions. The peak accelerations in the X, Y, and Z directions are 0.60 g, 0.45 g, and 0.60 g, respectively, where *g* is the gravity acceleration. The NM program is used to analyze the seismic dynamic response of the wedge, and the curves of the motion mode, velocity, safety factor, and displacement over time are illustrated in Figure 8. The results show that the motion mode of the wedge is double faces sliding before 7.65 seconds, and then it is converted into single face sliding, where 0 denotes free falling, 1 denotes single face sliding, 2 denotes double faces sliding, and 3 denotes the static; the velocity reaches a maximum of 0.42 m/s at 14.78 seconds; the safety factor reaches a minimum of 0.22 at 14.93 seconds, where the safety factor takes 3 when its value exceeds 3; the permanent displacement is 0.34 m.

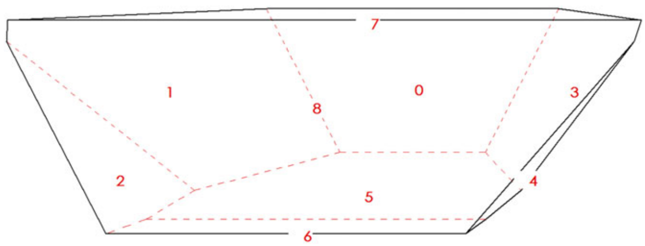

FIGURE 9 Geometric shape of the slope block [Colour figure can be viewed at wileyonlinelibrary.com]

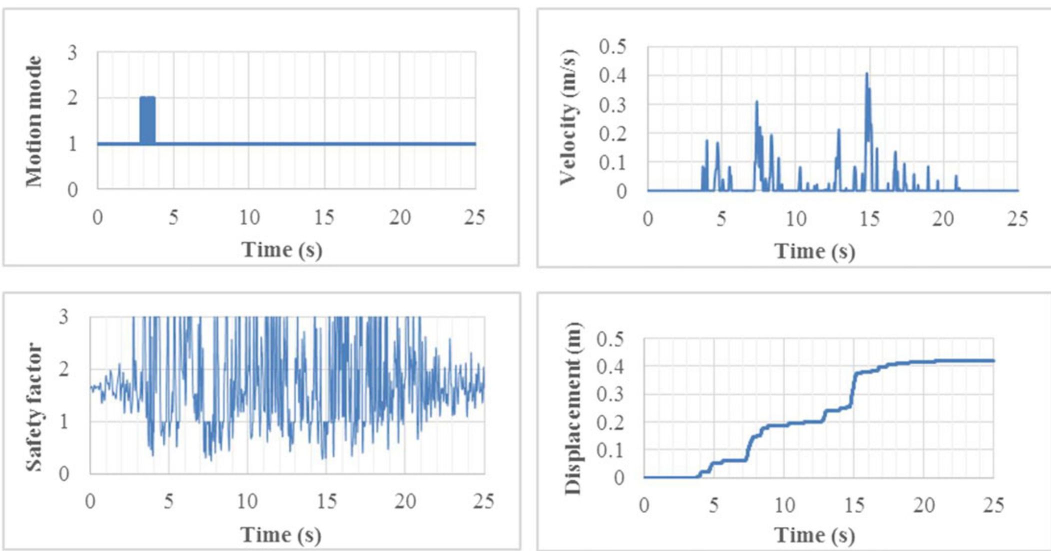

FIGURE 10 Seismic dynamic response of the slope block [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 4 The geometric and mechanical parameters of the structural and free planes for the complex rock block

Structural and Free Planes	α (°)	β (°)	A Point x_0, y_0, z_0 (m)	Friction Angle (°)	Cohesion Force (Kpa)	Area (m^2)
0	75	180	0, 0, 0	40	0	130.29 (A) /28.87 (B)/0.86 (C)
1	30	105	−24, 0, 0	25	0	110.57 (A) /91.80 (B) /217.26 (C)
2	40	245	6, 0, 0	30	0	91.67 (A) /76.10 (B) /180.11 (C)
3	0	0	0, 0, 0	-	0	263.50 (A)
4	45	180	0, −8, 0	-	0	207.39 (A)
5	5	0	0, 0, −6	-	0	211.24 (A) /267.43 (B)
6	55	180	0, −16, −6	-	0	67.79 (B)
7	10	0	0, 0, −10	-	0	162.50 (B) /259.33 (C)
8	60	180	0, −23, −10	-	0	141.92 (C)

In fact, the seismic dynamic response analysis of the rock block is very dependent on the values of the friction angle and cohesion force of the sliding face.[48] When the friction angles of two structural planes are 33°, and the cohesion forces of two structural planes are 19.6 Kpa, the wedge keep static under the ground motion action, and no permanent displacement is produced.

5.3 | A rock block of slope

In the slope example, a movable block is formed by cutting the rock with seven structural planes 0, 1, 2, 3, 4, 5, and 6 and two free planes 7 and 8. The geometric and mechanical parameters of the structural and free planes are shown in Table 3. The density of the rock is 2400 kg/m^3, and the gravity acceleration is 9.8 m/s^2. The digital coding is

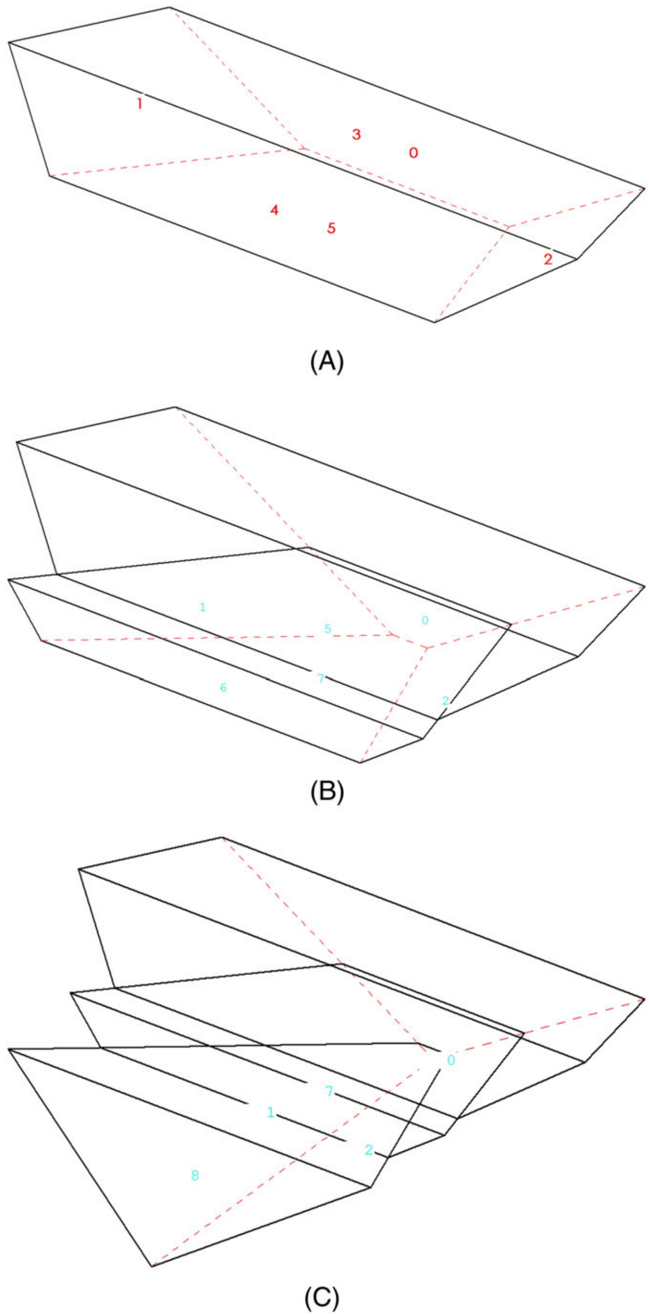

FIGURE 11 Geometric shape of the combined rock block A, subblock A; B, subblock A and B; and C, subblock A, B, and C [Colour figure can be viewed at wileyonlinelibrary.com]

000000011, Figure 9 shows the rock block, and its volume is 100.37m^3. The motion mode of the wedge is single face sliding because of its own weight, and $\vec{s} = (0, -0.951, -0.309)$, $S = 0.729$MN, $T = 1.194$MN, $N_1 = 2.245$MN, and $F = 1.638$.

The ground motion accelerations shown in Figure 7 are inputs to analyze the seismic dynamic response of the slope block, and the curves of the motion mode, velocity, safety factor, and displacement over time are illustrated in Figure 10. It shows that the motion mode of the rock block is mainly single face sliding, occasionally converts to double faces sliding; the velocity reaches a maximum of 0.40 m/s at 14.80 seconds, the safety factor reaches a minimum of 0.24 at 7.67 seconds, and the permanent displacement is 0.42 m.

5.4 | A combined rock block

In this example, three convex subblocks A, B, and C are formed by cutting the rock with three structural planes 0, 1, and 2, and six free planes 3, 4, 5, 6, 7, and 8, and they combine to form a complex rock block. The geometric and mechanical parameters of the structural and free planes are shown in Table 4. The density of the rock is 2400 kg/m^3, and the gravity acceleration is 9.8 m/s^2. The digital coding of subblock A is 000110222, and its volume is 1345.97m^3. The digital coding of subblock B is 000221102, and its volume is 662.36m^3. The digital coding of the subblock is 000222211, and its volume is 903.74m^3. The areas of the polygonal faces of each convex subblock are listed in Table 4. However, some structural or free planes are the common planes of the subblocks, and there may be two or more polygonal faces in these common planes. For example, all the convex subblocks contain the structural plane 0, and three polygonal faces are formed, and their areas are 130.29 (A) m^2, 28.87 (B) m^2, and 0.86 (C) m^2, respectively, here, 130.29 (A) m^2 means that the area of the structural plane 0 in subblock A is 130.29 m^2; the free plane 5 is the common plane of the convex subblocks A and B, and two polygonal faces are formed, and their areas are 211.24 (A) m^2 and 267.43 (B) m^2, respectively.

Figure 11 shows the convex subblocks and the combined rock block, The motion mode of the combined rock block is double faces sliding due to its own weight, and $\vec{s} = (0.150, -0.962, -0.227)$, $S = 15.569$MN, $T = 40.507$MN, $N_1 = 45.313$MN, $N_2 = 33.563$MN, and $F = 2.602$.

The ground motion accelerations shown in Figure 7 is input to analysis the seismic dynamic response of the combined rock block, and the curves of the motion mode, velocity, safety factor, and displacement over time are illustrated in Figure 12. It shows that the motion mode of the complex rock block is double faces sliding before 7.64 seconds, and then it is converted into single face sliding; the velocity reaches a maximum of 0.37 m/s at 14.79 seconds, the safety factor reaches a minimum of 0.19 at 17.94 seconds, and the permanent displacement is 0.22 m.

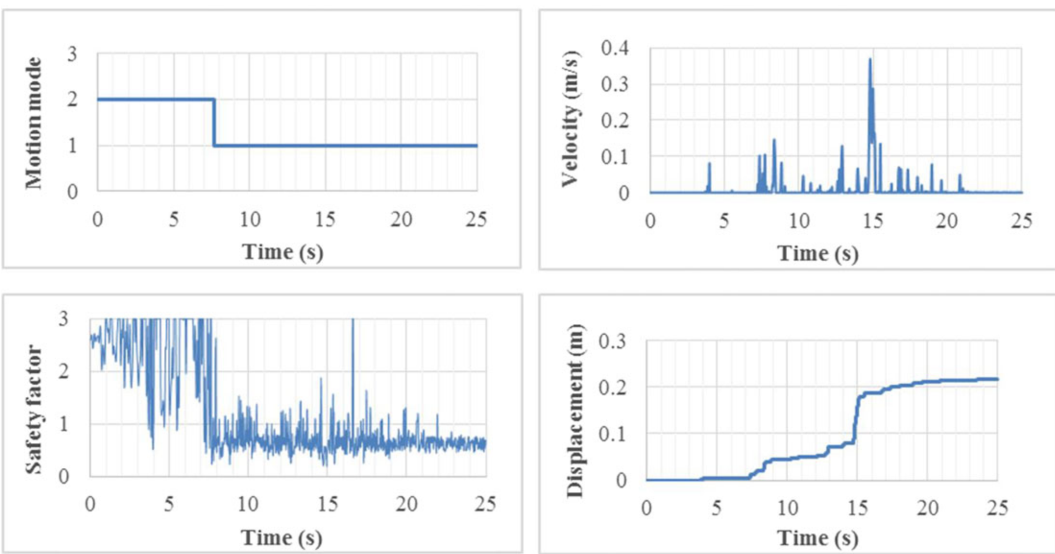

FIGURE 12 Seismic dynamic response of the combined rock block [Colour figure can be viewed at wileyonlinelibrary.com]

6 | CONCLUSIONS

The Newmark analysis is a method between simplistic pseudo-static analysis and sophisticated finite element modeling. In this paper, a general method combining the Newmark method and the block theory is introduced to analyze the seismic dynamic response of a rock block, and the motion mode, velocity, safety factor, and the displacement are effectively estimated, which provides an easy way to quantitatively evaluate the dynamic stability analysis of a rock block.

The method of forming a three-dimensional rock block is the premise work before the seismic dynamic analysis. A rock block may be convex or concave, and is formed by cutting the rock with several structural and free planes. The digital coding method is introduced to represent the spatial relationship between the rock block and these planes. When the rock block is concave, the combination method is employed to divide the complex rock block into several convex subblocks. For the geometric description of a convex block or subblock, it is obtained through the analysis of the digital coding and the occurrence parameters of the structural and free planes. First, all the vertices of the rock block are obtained by calculating the intersection among arbitrary three planes. Second, the vertices in each plane are arranged in order to identify all the edges of a polygonal face. Last, a topological relationship among block-polygon face-edge-vertex is formed, and the volume of the rock block and the areas of all polygonal faces can be analytically calculated with the simplex integration method.

The general method introduced in this paper should satisfy the assumptions of both the block theory and the Newmark method. The assumptions mainly include the rock block is a rigid body, the structural planes are infinite, the motion mode is translation, and the displacement in a time step is small. In the general method, first, the geometric description of the rock block, mechanical parameters of the structural and free planes, and the ground motion record need to be input. Second, the dynamic analysis is carried out in time step. In each time step, the key technologies including analysis of the seismic force and motion mode, trial of the incremental displacement, check for the block entrance, and update of the motion parameters are performed in order. Last, the curves of the motion mode, velocity, safety factor, and displacement over time can be output.

Comparing the simulation results and the theoretical solution, two verification examples have proved the correctness of the general method. Three examples, including a wedge, a rock block of slope, and a combined rock block, are used to analyze the practicality of the general method. In these examples, first, the areas of the polygonal faces and the volume of the rock block are obtained using the block formation method. Second, the motion mode, sliding force, resistant force, normal force, and the safety factor of the rock block because of its own weight are calculated using the traditional block theory. Last, for a given ground motion record, the Newmark program is used to analysis the seismic dynamics response of the rock block, and the permanent displacement of the rock block after the earthquake action is obtained, which provides a quantitative parameter to evaluate the dynamic stability analysis of the rock block.

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