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Unified Overload Method of Slope Stability Analysis Based on Potential Sliding Direction

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Abstract

The overload method is difficult to be promoted in slope stability analysis for its disunity of loading forms and directions. Based on the traditional overload method and the Strength Reduction Method (SRM) in which the limit equilibrium state of the slope was reached by reducing sliding resisting force without changing the sliding force, a new way to reach the limit equilibrium state of the slope was developed by increasing sliding force without changing resisting force. Referring the loading forms in Gravity Increase Method (GIM) and sliding direction determination in Vector Sum Method (VSM), the theoretical relationship was built between overload coefficient and safety factor of Vector Sum Method (VSM) and the unified overload method based on overall potential sliding direction was proposed. The loading forms and directions were unified by this method, respectively. Sliding surface could be determined while solving the safety factor and the developing direction of overload method in slope stability analysis application was indicated. Three representative slopes with fixed sliding surfaces and two slopes with unknown sliding surfaces were taken as examples to compare results from Limit Equilibrium Method (LEM), Strength Reduction Method (SRM), Vector Sum Method (VSM), Gravity Increase Method (GIM) and overloading method along the horizontal direction with each other. The safety factor resulted from the method proposed in this paper was close to the one from Vector Sum Method (VSM) and the location of sliding surface was close to the one from Strength Reduction Method (SRM). Thus the reliability of the method was testified.

Keywords: *unified overload method, safety factor, potential sliding direction, vector sum method*

1. Introduction

Slope stability analysis is the most important theoretical and practical problem in geotechnical engineering and a problem which is still unsolved perfectly in classic soil mechanics. Common methods at present include Limit Equilibrium Method (LEM), Strength Reduction Method (SRM), overload method, Finite Element Limit Equilibrium Method (FE-LEM), Vector Sum Method (VSM), etc. With Limit Equilibrium Method (LEM), the limit equilibrium state of overall sliding surface is reached by reducing the sliding resisting force basing on Mohr-Coulomb law, and the safety factor is obtained by balancing the forces and moments in the soil slices. The different limit equilibrium methods are proposed based on the different assumptions in the distribution of internal forces (Janbu, 1954; Bishop, 1955; Morgenstern *et al.*, 1965; Spencer, 1967). These methods are widely accepted by geotechnical engineer for its simplicity and convenience. However, researchers gradually found that the internal stress-strain relationship was not taken into account in the Limit Equilibrium Method (LEM) (Shao *et al.*, 2011). In Strength Reduction Method (SRM), the loads on slope are fixed and the soil strength parameters are reduced to make the slope reach limit equilibrium state. Then the strength reduction coefficient is considered as the slope safety factor (Zienkiewicz *et al.*, 1975; Griffiths *et al.*, 1999). With Strength Reduction Method (SRM), the potential sliding surface and safety factor can be determined quickly while questions are being raised about the reduction parameter (Manzari *et al.*, 2000; Zhang *et al.*, 2004; Zheng *et al.*, 2005; Yang *et al.*, 2009), the reduction range (Xue *et al.*, 2011; Chen *et al.*, 2013), the way to reducing parameters (Tang *et al.*, 2007; Yuan *et al.*, 2013; Zhao *et al.*, 2014) and the failure criterions (Ugai, 1989; Matsui *et al.*, 1992; Liu *et al.*, 2005; Zhao *et al.*, 2005; Wan *et al.*, 2010). In overload method, the slope soil strength parameters are fixed and loads are increased gradually to make the slope reach limit equilibrium state. The safety factor can be derived by measuring the ratio of critical failure load to the normal load (Shao *et al.*, 2011). Because of different loading forms and directions in overload method, various overload factors and slope safety factors have been obtained by different researchers. This introduced difficulty into building relationship

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between those factors and the one recommended in the criterion. Then the popularization of overload method was impeded (Sarma *et al.*, 1973; Seo *et al.*, 1998; Swan *et al.*, 1999; Xu *et al.*, 2007; Liu *et al.*, 2008; Yang *et al.*, 2009; Li *et al.*, 2009). In finite element limit equilibrium method (FE-LEM), Finite Element Method (FEM) was adopt to calculate the sliding resisting force and sliding force and the safety factor is the ratio of sliding resisting force to sliding force (Yin *et al.*, 2005; Liu *et al.*, 2015). For a straight slip line, the physical meaning of the safety factor is clear. For a circular slip line, the safety factor is actually the ratio of total resisting moment to total driving moment. For a non-straight line or a non-circular slip line, however, the physical meaning of the safety factor is questioned by some researchers because the integration in the definition of this safety factor is neither the summation of force vectors in space nor the summation of projections of force vectors in a fixed direction. In Vector Sum Method (VSM), assisted by stress field calculated by Finite Element Method (FEM), the sliding resisting force vectors and sliding force vectors can be obtained, and the safety factor was determined by the ratio of the projections of vector sum of sliding resisting forces to that of sliding forces in the potential sliding direction. Comparing with other methods, the Vector Sum Method (VSM) has more strict physical meaning (Ge *et al.*, 1995; Liu *et al.*, 2007; Ge *et al.*, 2008; Guo *et al.*, 2009). With Finite Element Limit Equilibrium Method (FE-LEM) and Vector Sum Method (VSM), the slope safety factor can be determined precisely while optimal algorithm must be involved in searching sliding surface. This may result in great trouble for engineers (Zhang *et al.*, 2006). Concluding from previous analysis, it can be found that the reason for overload method not being promoted widely are as follows: ① it is hard for researchers to differentiate the contribution of overload to the sliding resisting force and sliding force, ② it is impossible to define reasonable unified criterion for loading direction.

For the first reason, the main point of Strength Reduction Method (SRM) is to reach limit equilibrium state by reducing sliding resisting force without changing sliding force. In normal overload method, sliding force and resisting force are increased simultaneously, and it is hard to separate the contribution of overload to the sliding resisting force and sliding force. The question is whether there is a method with which we can increase sliding force without changing the sliding resisting force to reach the slope limit equilibrium state. For the second reason, it seems that we can solve it by referring the overall potential sliding direction in Vector Sum Method (VSM). Suppose there is a force acting on the potential sliding direction. Then this force will contribute to sliding force only and then the contribution of external force to sliding resisting force and sliding force can be separated clearly. A developed overload method corresponding to Strength Reduction Method (SRM) is formed. With this method slope safety factor and the sliding surface can be determined at the same time.

In this paper, referring the slope overall potential sliding direction in Vector Sum Method (VSM), the way to reach slope limit equilibrium state in Strength Reduction Method (SRM) and overload form in Gravity Increase Method (GIM), unified overload method were developed based on normal overload method by increasing sliding force without changing sliding resisting force. With this method, the loading forms and directions were unified respectively, and safety factor and sliding surface can be determined simultaneously. As a result, application of overload method in slope stability analysis is improved.

2. Principle of Unified Overload Method of Slope Stability Analysis Based on Potential Sliding Direction

2.1 Relationship between Overload Factor and Safety Factor of Vector Sum Method

The safety factor of vector sum method is the ratio of the projections of vector sum of sliding resisting forces to that of sliding forces in the potential sliding direction. Both resisting force vector and sliding force vector include the contribution of normal stress and the potential sliding direction is determined by the direction of the vector sum of resisting shear stress acting on potential sliding surface (Guo *et al.*, 2009). Suppose the direction of the vector sum of resisting shear stress acting on potential sliding surface is the overall sliding direction. Extra force acting along the overall sliding direction can only increase the sliding force and the effects of load on resisting sliding force and sliding force can be separated. Basing on the analysis, the relationship expression between overload coefficient and safety factor of vector sum method is deduced as follows.

The slope safety factor of vector sum method at any moment is

$$F_s = \frac{\int_s \left[\boldsymbol{\sigma}_s' \cdot (-\boldsymbol{d})\right] ds}{\int_s \left(\boldsymbol{\sigma}_s \cdot \boldsymbol{d}\right) ds} \tag{1}$$

Where, $\boldsymbol{\sigma}_s$ is the sliding stress vector at point A on potential sliding surface, $\boldsymbol{\sigma}_s'$ is the sliding resisting stress vector at point A on potential sliding surface, $\boldsymbol{d}$ is a vector in the overall sliding direction which is determined by the limit sliding resisting shear stress of the points on sliding surface (Guo *et al.*, 2009), and S is the potential sliding surface. Directions of these parameters are shown in Fig. 1.

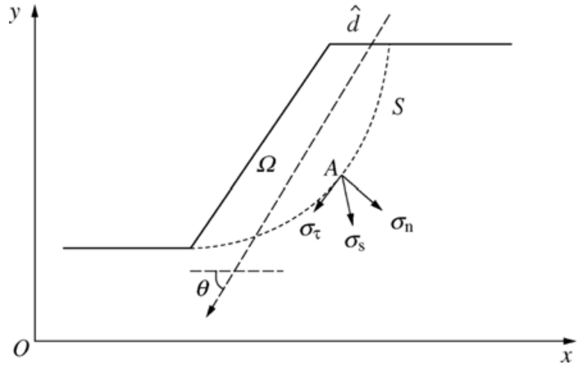

Fig. 1. Safety Factor by Two-dimensional Vector Sum Method

Considering the slope stability only effected by self-weight, from static equilibrium condition we can obtain

$$\int_s (\sigma_s \cdot \boldsymbol{d})\, ds = mg\sin\theta \tag{2}$$

Where, m is the mass of sliding body, g is the gravity acceleration, θ is the angle between potential sliding direction and horizontal.

Accordingly, impose a force of *Kmg* on the overall potential sliding direction to make the slope reach limit equilibrium state and base on the equilibrium between sliding resisting force and sliding force, then

$$\int_s \left[\sigma_s' \cdot (-\boldsymbol{d})\right] ds = Kmg + mg\sin\theta \tag{3}$$

Where K is overload coefficient. Because this force is imposed in the potential sliding direction, it has no impact on sliding resisting force. This means that the projection of vector sum of resisting forces in overall sliding direction is equal to that of the limit equilibrium state. Substitute

Equation (3) into Eq. (1), then

$$F_s = \frac{Kmg + mg\sin\theta}{\int_s (\sigma_s \cdot \boldsymbol{d})\, ds} \tag{4}$$

Substitute Eq. (2) into Eq. (4), then

$$F_s = \frac{mg\sin\theta + Kmg}{mg\sin\theta} = 1 + \frac{K}{\sin\theta} \tag{5}$$

The relationship expression between overload coefficient and safety factor of vector sum method is established. Especially, even Eq. (5) is developed in the condition of only self-weight is concerned in slope stability analysis. It is suitable for any other conditions in slope stability analysis. Stability analysis concerning seismic load with quasi-static method is one of the cases.

$$F_s = \frac{mg\sin\theta + ma_h\cos\theta + Kmg}{mg\sin\theta + ma_h\cos\theta} = 1 + \frac{K}{\sin\theta + a_h\cos\theta/g} \tag{6}$$

Where a_h is the seismic acceleration in the horizontal direction.

2.2 Implement of Unified Overload Method of Slope Stability Analysis Based on Potential Sliding Direction

With the expression of the safety factor of vector sum method can be derived by Eq. (5). The direction of load in unified overload method is determined by limit resisting shear stress of points on the sliding surface. According to the Mohr-Coulomb yield criterion, the overall potential sliding direction of slope can be calculated by Eqs. (7) and (8).

$$\boldsymbol{d} = \frac{-\int_s \left[(c - \sigma_n\tan\varphi)\boldsymbol{d}_r\right] ds}{\left\| \int_s \left[(c - \sigma_n\tan\varphi)\boldsymbol{d}_r\right] ds \right\|} \tag{7}$$

$$\theta = \arctan\left|\frac{d_y}{d_x}\right| \tag{8}$$

Where c is the cohesion, φ is the friction angle, $\boldsymbol{d}_r$ is the unit shear stress vector at any point on the potential sliding surface, d_x and d_y are the x-component and y-component of $\boldsymbol{d}$, respectively.

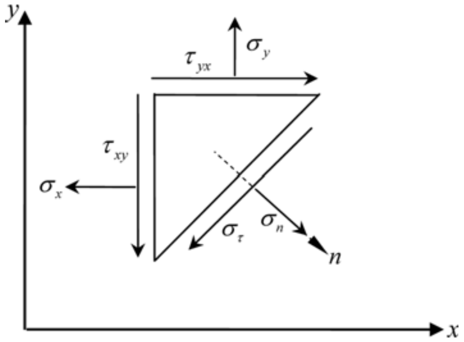

Fig. 2. Normal and Shear Stresses on an Inclined Plane at a Point in a Soil

For slope stability analysis on fixed sliding surface, the distribution of stress field can be calculated by Finite Element Method (FEM). The distribution of stress state at any point on the potential sliding surface can be calculated by Eqs. (9) and (10). These parameters in Eqs. (9) and (10) are shown in Fig. 2, and the overall potential sliding direction can be obtained by Eqs. (7) and (8). In this case, the unified overload method of slope stability analysis based on potential sliding direction can be implemented conveniently.

$$\sigma_n = l^2\sigma_x + m^2\sigma_y + 2lm\tau_{xy} \tag{9}$$

$$\sigma_\tau = lm(\sigma_x - \sigma_y) + (m^2 - l^2)\tau_{xy} \tag{10}$$

Where $\boldsymbol{n}$ is unit normal vector at any point on potential sliding surface, $l = \cos(n,x)$ and $m = \cos(n,y)$ are direction cosines of outer normal vector at any point on potential sliding surface, σ_x, σ_y and τ_{xy} are the x-direction stress, y-direction stress and x-y shear stress at any point on potential sliding surface, respectively, σ_n and σ_τ are the normal stress and shear stress at any point on potential sliding surface, respectively.

For slope stability analysis on unknown sliding surface, iteration must be adopted. In this case, the implement of the unified overload method of slope stability analysis based on potential sliding direction mainly includes the following steps.

(1) The limit equilibrium state of the slope is reached by gravity increase method (overload on vertical direction) (Yang *et al.*, 2009; Li *et al.*, 2009) and the sliding surface is determined by the maximum equivalent plastic strain. The equivalent plastic strain is obtained by integrating the equivalent plastic strain rate:

$$\varepsilon_{eq}^p = \int_0^t \sqrt{2\dot{\varepsilon}_{ij}^p \dot{\varepsilon}_{ij}^p / 3}\, dt \tag{11}$$

Shown in Fig. 3 is the potential sliding surface of the slope determined by the maximum equivalent plastic strain. We arrange a group of vertical lines along the horizontal direction. Then we can find out the point with the maximum equivalent plastic strain (solid point) on each vertical line. In this way, we can obtain a set of maximizers (x_i, y_i), $i = 1,\ldots, n$. These maximizers compose a set of good functional data. We can find a piece-wise smooth curve that approaches the functional data in the least squares. The resulting curve is the sliding surface of the slope

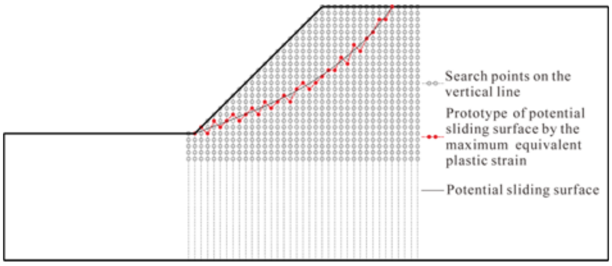

Fig. 3. Potential Sliding Surface of the Slope Determined by the Maximum Equivalent Plastic Strain

(Zheng *et al.*, 2009).

(2) It can be calculated that the sliding resisting shear stress of points on sliding surface, and compose all the resisting shear forces on the sliding surface. The direction of the resultant force vector is used as the loading direction in the consequent overload method. This process is iterated until the loading direction of previous step matching the direction of vector sum of the sliding resisting shear forces. The corresponding sliding surface then is the final sliding surface and the safety factor can be calculated by overload coefficient with Eq. (5).

3. Illustrative Examples of Fixed Sliding Surface and Method Verification

Three cases, i.e., homogeneous slope, slopes with two layers and three layers, have been selected to verify the reliability of the method. Let us compare the results from the proposed method in the paper with those from Limit Equilibrium Method (LEM) and Vector Sum Method (VSM). The sliding surface was determined by limit equilibrium method (Morgenstern *et al.*, 1965). The stress field was calculated by finite element analysis software ABAQUS.

3.1 Example 1: A Homogeneous Slope

The example of the homogeneous slope originates from Liu (Liu *et al.*, 2005). The slope height is 20 m with a slope ratio of 1: 1. The distance from the slope toe to the front of the model is 30 m. The distance from the slope crest to the back of the model is 55 m. The total height of the model is 40 m. The total width of the model is 105 m. The bottom boundary condition is pinned, and rollers are used along the vertical extents of the model. All conditions are shown in Fig. 4. Ideal elastoplastic constitutive model with Mohr-Coulomb failure criterion and non-associated flow rule is used in the finite element analysis. Material parameters are shown in Table 1.

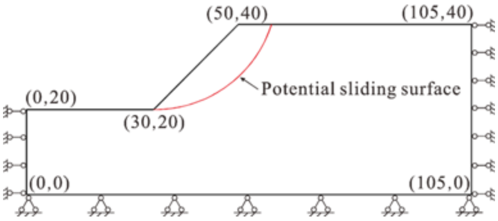

Fig. 4. Geometry and Boundary Conditions for the Homogeneous Slope with a Fixed Sliding Surface

Table 1. Material Parameters for Numerical Simulation of the Homogeneous Slope

Unit weight /(kN/m^3)	Young's modulus /kPa	Poisson's ratio	Cohesion /kPa	Friction angle /°
20	1×10^5	0.3	42	17

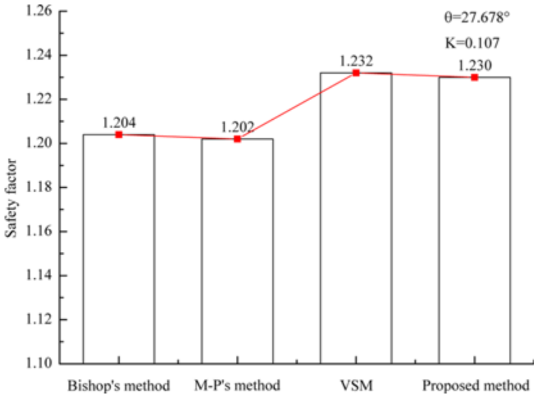

Fig. 5. Results from Different Methods for the Homogeneous Slope

For the homogeneous slope, results from Limit Equilibrium Method (LEM), Vector Sum Method (VSM) and the proposed method in the paper are shown in Fig. 5. The sliding angle is 27.678° and overload coefficient is 0.107 determined by the proposed method. This means that the slope reaches the limit equilibrium state when imposing a force of *0.107 mg* along the overall potential sliding direction. The slope safety factor is 1.230 calculated by Eq. (5). By Vector Sum Method (VSM), the safety factor is 1.232 which has a difference of 0.002 from the proposed method. This indicates the feasibility of the proposed method. Besides, the safety factors resulted from both proposed method and vector sum method are larger than that from Bishop's method and M-P's method, and this is in agreement with the results from Guo (Guo *et al.*, 2010).

3.2 Example 2: A Slope with Two Layers

The example of the two-layer slope originates from Zheng (Zheng *et al.*, 2006). The distance from the slope toe to the front of the model is 50 m. The distance from the slope crest to the back of the model is 50 m. The total height of the model is 58 m.

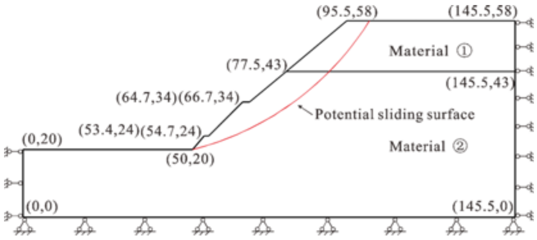

Fig. 6. Geometry and Boundary Conditions for the Two-layer Slope with a Fixed Sliding Surface

Table 2. Material Parameters for Numerical Simulation of the Two-layer Slope

Material	Unit weight /(kN/m^3)	Young's modulus /kPa	Poisson's ratio	Cohesion /kPa	Friction angle /°
①	24.0	2×10^7	0.35	34.0	26
②	25.0	5×10^7	0.30	39.0	35

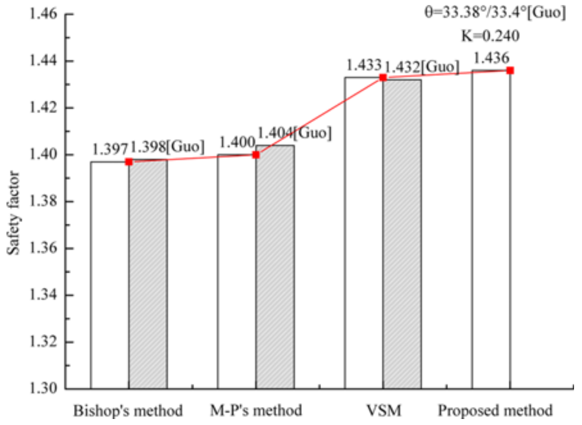

Fig. 7. Results from Different Methods for the Two-layer Slope

The bottom boundary condition is pinned, and rollers are used along the vertical extents of the model. All conditions are shown in Fig. 6. Ideal elastoplastic constitutive model with Mohr-Coulomb failure criterion and non-associated flow rule is used in the finite element analysis. Two materials are involved and all parameters are shown in Table 2.

For the two-layer slope, results from Limit Equilibrium Method (LEM), Vector Sum Method (VSM) and the proposed method in the paper are shown in Fig. 7. Specifically, the results from different method are in agreement with the results from Guo (Guo *et al.*, 2009), and this indicates the reliability of the results from different method in this paper. The sliding angle is 33.38° and overload coefficient is 0.240 determined by the proposed method. This means that the slope reaches the limit equilibrium state when imposing a force of *0.240 mg* along the overall potential sliding direction. The slope safety factor is 1.436 calculated by Eq. (5). By Vector Sum Method (VSM), the safety factor is 1.433 which has a difference of 0.003 from the proposed method. This indicates the feasibility of the proposed method in two-layer slope stability analysis. Comparing the safety factor from the proposed method with those from Bishop's method and M-P's method, it can be found that the latter one is smaller. This can be rationally explained. In the system of slices of the limit equilibrium methods, in order to render the slice system determinate, some assumptions on the interaction between slices have to be made. These assumptions might more or less loose the constraints between the slices and hence the degree of safety of the slope might be underestimated due to the assumption of such a rigid system.

3.3 Example 3: A Slope with Three Layers

Table 3. Material Parameters for Numerical Simulation of the Three-layer Slope

Material	Unit weight /(kN/m^3)	Young's modulus/kPa	Poisson's ratio	Cohesion /kPa	Friction angle /°
①	19.5	1×10^4	0.25	0.0	38.0
②	19.5	1×10^4	0.25	5.3	23.0
③	19.5	1×10^4	0.25	7.2	20.0

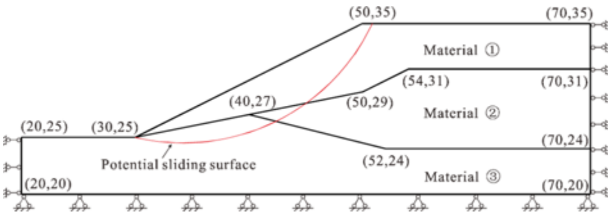

Fig. 8. Geometry and Boundary Conditions for the Three-layer Slope with a Fixed Sliding Surface

The example of the three-layer slope originates from Chen (Chen *et al.*, 2003). The slope height is 10 m with a slope ratio of 1: 2. The distance from the slope toe to the front of the model is 10 m. The distance from the slope crest to the back of the model is 20 m. The total height of the model is 15 m. The bottom boundary condition is pinned, and rollers are used along the vertical extents of the model. All conditions are shown in Fig. 8. Ideal elastoplastic constitutive model with Mohr-Coulomb failure criterion and non-associated flow rule is used in the finite element analysis. Three materials are involved and all parameters are shown in Table 3.

For the three-layer slope, results from Limit Equilibrium Method (LEM), Vector Sum Method (VSM) and the proposed method in the paper are shown in Fig. 9. The sliding angle is 20.66° and overload coefficient is 0.147 determined by the proposed method. This means that the slope reaches the limit equilibrium state when imposing a force of *0.147 mg* along the overall potential sliding direction. The slope safety factor is 1.417 calculated by Eq. (5). By Vector Sum Method (VSM), the safety factor is 1.411 which has a difference of 0.006 from the proposed method, and this indicates the feasibility of the proposed method in the three-

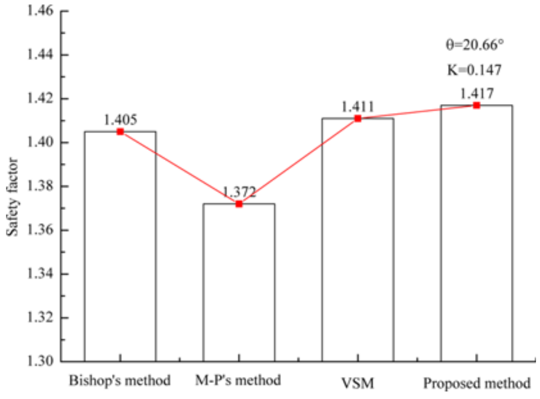

Fig. 9. Results from Different Methods for the Three-layer Slope

layer slope stability analysis. Comparing the safety factor from the proposed method with that from traditional limit equilibrium method, it turns out that the result from Bishop's method is close to the proposed method and vector sum method while result from M-P's method is smaller than the proposed method and vector sum method.

From the analysis, it can be found that safety factors from the proposed method for three cases are close to that from vector sum method with the largest error of 0.006. Hence the verification of the proposed method has been completed.

4. Illustrative Example of Unknown Sliding Surface and Method Verification

Previous analysis about fixed sliding surface slope shows the feasibility of proposed method in the slope stability analysis. By studying on unknown sliding surface slope, this section will explain the feasibility of proposed method in both sliding surface determination and safety factor calculation. Verification of proposed method is accomplished by comparing the results from proposed method with that from Limit Equilibrium Method (LEM), Strength Reduction Method (SRM) and Vector Sum Method (VSM).

4.1 Example 1: A Homogeneous Slope

The homogeneous slope originates from Wan (Wan *et al.*, 2010). The slope height is 10 m with a slope ratio of 1: 2. The distance from the slope toe to the front of the model is 20 m. The distance from the slope crest to the back of the model is 20 m. The total height of the model is 20 m. The bottom boundary condition is pinned, and rollers are used along the vertical extents of the model. All conditions are shown in Fig. 10. Ideal elastoplastic constitutive model with Mohr-Coulomb failure criterion and non-associated flow rule is used in the finite element analysis. Material parameters are shown in Table 4.

Table 4. Material Parameters for Numerical Simulation of the Homogeneous Slope

Unit weight /(kN/m^3)	Young's modulus /kPa	Poisson's ratio	Cohesion /kPa	Friction angle /°
20	1×10^5	0.3	10	20

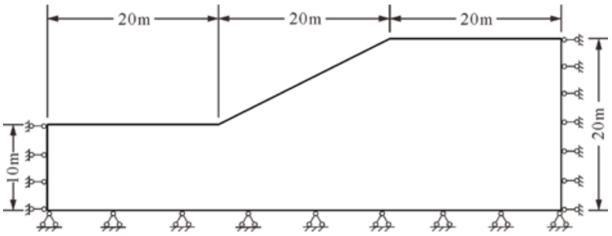

Fig. 10. Geometry and Boundary Conditions for the Homogeneous Slope

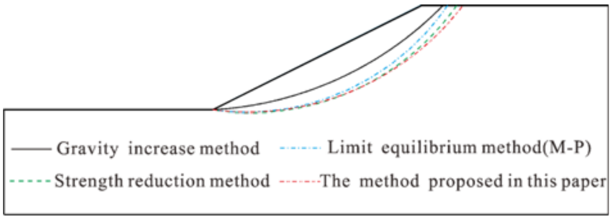

Fig. 11. Potential Sliding Surfaces using Different Methods for the Homogeneous Slope

Zheng (2006; 2009) points out that it is reasonable to solve the safety factor calculation and critical sliding surface determination by Strength Reduction Method (SRM). Take the safety factor and the critical sliding surface determined by Strength Reduction Method (SRM) as standards, comparative analysis of the results from Gravity Increase Method (GIM), limit equilibrium method (M-P) with the proposed method is conducted in order to verify the proposed method. Shown in Fig. 11 is that all the critical sliding surfaces obtained using different methods passed through the toe of slope. This is consistent with the common understanding of the failure behavior of a homogeneous slope. However, the locations of those sliding surfaces are significantly different. The one from Gravity Increase Method (GIM) is shallower than the others. The one from limit equilibrium method (M-P) is located between the one from Gravity Increase Method (GIM) and the one from Strength Reduction Method (SRM). The one from the proposed method is close to the one from Strength Reduction Method (SRM), and this means the result from the proposed method is reliable.

For the homogeneous slope, results from Limit Equilibrium Method (LEM), Vector Sum Method (VSM) and the proposed method in the paper are shown in Fig. 12. The sliding angle is 19.92° and overload coefficient is 0.138 determined by the proposed method. This means that the slope reaches the limit equilibrium state when imposing a force of *0.138 mg* along the overall potential sliding direction. The slope safety factor is 1.405 calculated by Eq. (5). By Vector Sum Method (VSM), the safety factor is 1.401 which has a difference of 0.004 from the proposed method. This indicates the feasibility of the proposed method. Besides, comparing the safety factor resulted from the proposed method with those from Bishop's method, M-P's

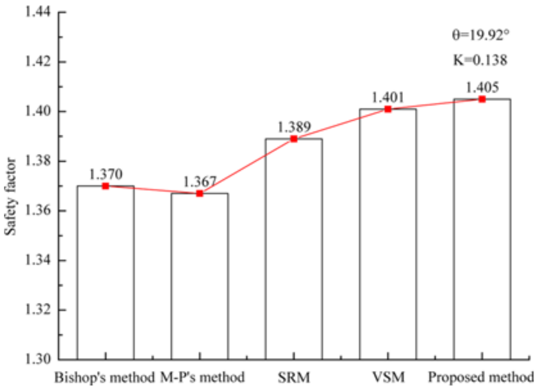

Fig. 12. Results from Different Methods for the Homogeneous Slope

method and Strength Reduction Method (SRM), it turns out that the safety factor from the proposed method is larger than the one from Strength Reduction Method (SRM), which is larger than those from Bishop's method and M-P's method. But the largest relative error of them is only 2.7%.

4.2 Example 2: A Slope with Water Table

The slope with water table originates from the documentation of slope/w software (Geo-slope international ltd, 2008). The slope height is 20 m with a slope ratio of 1: 2. The distance from the slope toe to the front of the model is 30 m. The distance from the slope crest to the back of the model is 30 m. The total height of the model is 40 m. The bottom boundary condition is pinned, and rollers are used along the vertical extents of the model. The location of the water table is shown in Fig. 13. Ideal elastoplastic constitutive model with Mohr-Coulomb failure criterion and non-associated flow rule is used in the effective stress analysis. Material parameters are shown in Table 5.

Comparative analysis of the results from overloading method along the horizontal direction, limit equilibrium method (M-P) and Strength Reduction Method (SRM) with the proposed method is conducted in order to verify the proposed method. Shown in Fig. 14 is that the locations of all the critical sliding surfaces obtained using different methods are significantly different. The one from overloading method along the horizontal direction is not steeper than others. The upper half portion of the potential sliding surface from the proposed method is close to the ones from Strength Reduction Method (SRM) and from limit equilibrium method (M-P). The lower half portion of the potential sliding surface from the proposed method is located between the one from Strength Reduction Method (SRM) and the one from limit equilibrium method (M-P). These mean the result from the proposed method is reliable.

Table 5. Material Parameters for Numerical Simulation of the Slope with Water Table

Unit weight /(kN/m^3)	Young's modulus /kPa	Poisson's ratio	Cohesion /kPa	Friction angle /°
20	1×10^5	0.4	8	30

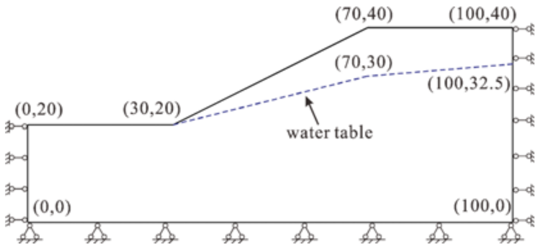

Fig. 13. Geometry and Boundary Conditions for the Slope with Water Table

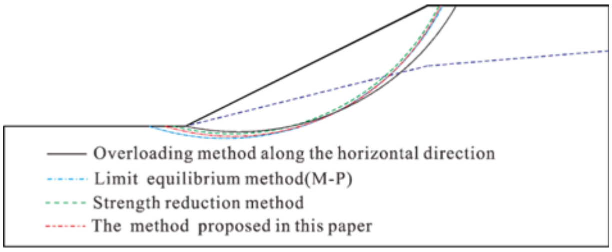

Fig. 14. Potential Sliding Surfaces using Different Methods for the Slope with Water Table

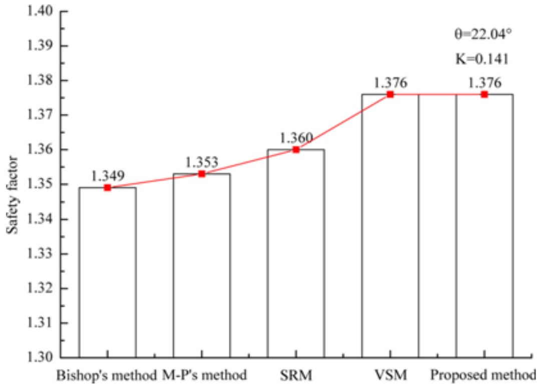

Fig. 15. Results from Different Methods for the Slope with Water Table

For the slope with water table, results from Limit Equilibrium Method (LEM), Vector Sum Method (VSM) and the proposed method in the paper are shown in Fig. 15. The sliding angle is 22.04° and overload coefficient is 0.141 determined by the proposed method. This means that the slope reaches the limit equilibrium state when imposing a force of *0.141 mg* along the overall potential sliding direction. The slope safety factor is 1.376 calculated by Eq. (5). By Vector Sum Method (VSM), the safety factor is 1.376 which is equal to that from the proposed method. This indicates the feasibility of the proposed method. Besides, comparing the safety factor resulted from the proposed method with those from Bishop's method, M-P's method and Strength Reduction Method (SRM), it turns out that the safety factor from the proposed method is larger than the one from Strength Reduction Method (SRM), which is larger than those from Bishop's method and M-P's method. But the largest relative error of them is only 2.0%.

From the points of sliding surface and safety factor, reliability of the proposed method is verified in the homogeneous slope stability analysis and the slope stability analysis with water table. In particular, it is more convenient to determine the slope safety factor and sliding surface simultaneously by the proposed method. This advantage is similar to strength reduction method, but is better than other methods.

5. Conclusions

In analogy to the loading form in Gravity Increase Method (GIM), referring to the thoughts of determining the overall sliding direction with Vector Sum Method (VSM) and reaching the limit equilibrium state by reducing sliding resisting force with Strength Reduction Method (SRM), basing on the overload method, the unified overload method of slope stability analysis which only increases the sliding force without changing the sliding resisting force was proposed. By this method, the slope safety factor and sliding surface can be determined at the same time, the loading forms and directions are unified. Hence the application of overload method in slope stability analysis was cleared. Following conclusions can be made.

1. Theoretical expression of overload coefficient basing on the overall sliding direction and the safety factor of vector sum method was established.
2. By comparing the results from Limit Equilibrium Method (LEM), Strength Reduction Method (SRM), Vector Sum Method (VSM), Gravity Increase Method (GIM) and overloading method along the horizontal direction with the proposed method, it can be found that the safety factor from the proposed method is nearly equal to that from Vector Sum Method (VSM) and the sliding surface is closed to that from Strength Reduction Method (SRM). Then the reasonability and reliability of the proposed method were verified.

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References

Bishop, A. W. (1955). “The use of slip circle in the stability analysis of slopes.” *Geotechnique*, Vol. 5, No. 1, pp. 7-17, DOI: 10.1680/geot.1955.5.1.7.

Chen, G. Q., Huang, R. Q., Zhou, H., Xu, Q., and Li, T. B. (2013). “Research on progressive failure for slope using dynamic strength reduction method.” *Rock and Soil Mechanics*, Vol. 34, No. 4, pp. 1140-1146, DOI: 10.16285/j.rsm.2013. 04.040.

Chen, Z. Y. (2003). *Soil Slope Stability Analysis: Theory · Methods and Programs*, China Water & Power Press, Beijing, P.R.China.

Ge, X. R. (2008). “Deformation control law of rock fatigue failure, real-time X-ray CT scan of geotechnical testing, and new method of stability analysis of slopes and dam foundations.” *Chinese Journal of Geotechnical Engineering*, Vol. 30, No. 1, pp. 1-20.

Ge, X. R., Feng, D. X., and Gu, X. R. (1995). “Stability and deformation analysis of complex rock foundations of several large dams and hydropower stations in China.” *Proc., International Workshop on Rock Foundation,* Rotterdam: A.A. Balkema, pp. 243-248.

Geo-slope International Ltd. (2008). *Stability modeling with slope/w 2007 version*, Calgary, Alberta, Canada.Web: http://www.geo-slope.com

Griffiths, D. V. and Lane, P. A. (1999). “Slope stability analysis by finite elements.” *Geotechnique*, Vol. 49, No. 3, pp. 387-403, DOI: 10.1680/geot.1999.49.3.387.

Guo, M. W. (2010). “Study on the Vector Sum Analysis Method of Slope and Dam Foundation Stability Against Sliding and Its Engineering Application.” *Institute of Rock and Soil Mechanics, Chinese Academy of Sciences*, Wuhan, P.R.China.

Guo, M. W., Ge, X. R., Li, C. G., Wang, S. L., and Liu, Y. Z. (2009). “Study on potential sliding direction in slope stability analysis based on vector sum method.” *Chinese Journal of Geotechnical Engineering*, Vol. 31, No. 4, pp. 577-583.

Janbu, N. (1954). “Application of composite slip surfaces for stability analysis.” *Proc., the European Conference on Stability of Earth Slopes*, Stockholm, Sweden, Vol. 3, pp. 43-49.

Li, L. C., Tang, C. A., Zhu, W. C., and Liang, Z. Z. (2009). “Numerical analysis of slope stability based on the gravity increase method.” *Computers and Geotechnics*, Vol. 36, No. 7, pp. 1246-1258, DOI: 10.1016/j.compgeo. 2009.06.004.

Liu, J., Zheng, T., and Li, J. L. (2008). “A comparative study of gravity reduction FEM and strength reduction FEM.” *China Civil Engineering Journal*, Vol. 41, No. 10, pp. 66-72, DOI: 10.15951/j.tmgcxb. 2008.10.014.

Liu, J. L., Luan, M. T., Zhao, S. F., Yuan, F. F., and Wang, J. L. (2005). “Discussion on criteria for evaluating stability of slope in elastoplastic FEM based on shear strength reduction technique.” *Rock and Soil Mechanics*, Vol. 26, No. 8, pp. 1345-1348, DOI: 10.16285/j.rsm. 2005.08.035.

Liu, S. Y., Shao, L. T., and Li, H. J. (2015). “Slope stability analysis using the limit equilibrium method and two finite element methods.” *Computers and Geotechnics*, Vol. 63, No. 1, pp. 291-298, DOI: 10.1016/j.compgeo. 2014.10.008.

Liu, Y. Z., Ge, X. R., Li, C. G., and Wang, S. L. (2007). “Stability analysis of analysis of slope and dam foundation based on vector method safety factor.” *Chinese Journal of Rock Mechanics and Engineering*, Vol. 26, No. 10, pp. 2130-2140, DOI: 10.3321/j.issn: 1000-6915.2007.10.025.

Manzari, M. T. and Mohamed, A. N. (2000). “Significance of soil dilatancy in slope stability analysis.” *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 126, No. 1, pp. 75-80, DOI: 10.1061/(ASCE)1090-0241(2000)126:1(75).

Matsui, T. and San, K.C. (1992). “Finite element slope stability analysis by shear strength reduction technique.” *Soils and Foundations,* Vol. 32, No. 1, pp. 59-70, DOI: 10.3208/sandf1972.32.59.

Morgenstern, N. R. and Price, V. E. (1965). “The analysis of the stability of general slip surfaces.” *Geotechnique*, Vol. 15, No. 1, pp. 79-93, DOI: 10.1680/geot.1965.15.1.79.

Sarma, S. K. (1973). “Stability analysis of embankments and slopes.” *Geotechnique*, Vol. 23, No. 3, pp. 423-433, DOI: 10.1680/geot. 1973.23.3.423.

Seo, Y. K. (1998). *Computational Methods for Elasto-plastic Slope Stability with Seepage*, the University of Iowa, Iowa, USA.

Shao, L.T. and Li, H. J. (2011). *Stability Analysis of Geotechnical Structures—Finite Element Method Limit Equilibrium Method and Its Application*, Science Press, Beijing, P.R.China.

Spencer, E. (1967). “A method of analysis of the stability of embankments assuming parallel inter-slice forces.” *Geotechnique*, Vol. 17, No. 1, pp. 11-26, DOI: 10.1680/geot.1967.17.1.11.

Swan, C. C. and Seo, Y. K. (1999). “Limit state analysis of earthen slopes using dual continuum/FEM approaches.” *International Journal of Numerical Analysis Method in Geotechnical*, Vol. 23, pp. 1359-1371, DOI: 10.1002/(SICI)1096-9853(199910)23:123.0.CO;2-Y.

Tang, F., Zheng, Y. R., and Zhao, S. Y. (2007). “Discussion on two safety factors for progressive failure of soil slope.” *Chinese Journal of*

Rock Mechanics and Engineering, Vol. 26, No. 7, pp. 1402-1407, DOI: 10.3321/j.issn:1000-6915.2007.07.013.

Ugai, K. (1989). "A method of calculation of total factor of safety of slopes by elastic-plastic FEM." *Soils and Foundations*, Vol. 29, No. 2, pp. 190-195, DOI: 10.3208/sandf1972.29.2_190.

Wan, S. S., Nian, T. K., Jiang, J. C., and Luan, M. T. (2010). "Discussion on several issues in slope stability analysis based on shear strength reduction finite element methods." *Rock and Soil Mechanics*, Vol. 31, No. 7, pp. 2283-2289, DOI: 10.3969/j.issn.1000-7598.2010.07.044.

Xu, W. Y., and Xiao, W. (2007). "Study on slope failure criterion based on strength reduction and gravity increase method." *Rock and Soil Mechanics*, Vol. 28, No. 3, pp. 0505-0511, DOI: 10.3969/j.issn.1000-7598.2007.03.014.

Xue, L., Sun, Q., Qin, S. Q., Liu, H. D., and Huang, X. (2011). "Scope of strength reduction for inhomogeneous slopes." *Chinese Journal of Geotechnical Engineering*, Vol. 33, No. 2, pp. 275-280.

Yang, G. H., Zhang, Y. C., and Zhang, Y. X. (2009). "Variable modulus elastoplastic strength reduction method and its application to slope stability analysis." *Chinese Journal of Rock Mechanics and Engineering*, Vol. 28, No. 7, pp. 1506-1512, DOI: 10.3321/j.issn:1000-6915.2009.07.026.

Yang, M. C. and Kang, Y. M. (2009). "Application of gravity increase method in slope stability analysis." *Journal of Harbin Institute of Technology,* Vol. 41, No. 6, pp. 0187-0190, DOI: 10.1109/TCSVT.2011.2162764.

Yin, Z. Z. and Lv, Q. F. (2005). "Finite element analysis of soil slope based on circular slip surface assumption." *Rock and Soil Mechanics,* Vol. 26, No. 10, pp. 1525-1529, DOI: 10.3969/j.issn.1000-7598.2005.10.001.

Yuan, W., Bai, B., Li, X. C., and Wang, H. B. (2013). "A strength reduction method based on double reduction parameters and its application." *Journal of Central South University,* Vol. 20, No. 9, pp. 2555-2562, DOI: 10.1007/s11771-013-1768-4.

Zhang, L. Y. and Zhang, J. M. (2006). "Extended algorithm using Monte Carlo techniques for searching general critical failure surface in slope stability analysis." *Chinese Journal of Geotechnical Engineering*, Vol. 28, No. 7, pp. 857-862, DOI: 10.3321/j.issn:1000-4548.2006.07.010.

Zhang, P. W. and Chen, Z. Y. (2004). "Finite element method for solving safety factor of slope stability." *Rock and Soil Mechanics*, Vol. 25, No. 11, pp. 1757-1760, DOI: 10.3969/j.issn.1000-7598.2004.11.016.

Zhao, L. H., Cao, J. Y., Tang, G. P., Wang, Z. B., and Tan, H. H. (2014). "Discussion on slope stability analysis with double strength reduction technique." *Rock and Soil Mechanics*, Vol. 35, No. 10, pp. 2977-2984, DOI: 10.16285/j.rsm.2014.10.019.

Zhao, S. Y., Zheng, Y. R., and Zhang, Y. F. (2005). "Study on slope failure criterion in strength reduction finite element method." *Rock and Soil Mechanics*, Vol. 26, No. 2, pp. 0332-0336, DOI: 10.3969/j.issn.1000-7598.2005.02.035.

Zheng, H., Liu, D. F., and Li, C. G. (2005). "Slope stability analysis based on elasto-plastic finite element method." *International Journal for Numerical Methods in Engineering*, Vol. 64, No. 14, pp. 1871-1888, DOI: 10.1002/nme.1406.

Zheng, H., Sun, G. H., and Liu, D. F. (2009). "A practical procedure for searching critical slip surfaces of slopes based on the strength reduction technique." *Computers and Geotechnics,* Vol. 36, Nos. 1-2, pp. 1-5, DOI: 10.1016/j.compgeo.2008.06.002.

Zheng, H., Tham, L. G., and Liu, D. F. (2006). "On two definitions of the factor of safety commonly used in the finite element slope stability analysis." *Computers and Geotechnics*, Vol. 33, No. 3, pp. 188-195, DOI: 10.1016/j.compgeo.2006.03.007.

Zienkiewicz, O. C., Humpheson, C., and Lewis, R. W. (1975). "Associated and non-associated visco-plasticity and plasticity in soil mechanics." *Geotechnique*, Vol. 25, No. 4, pp. 671-689, DOI: 10.1680/geot.1975.25.4.671.