

PAPER • OPEN ACCESS

Analysis of mechanical behavior of a pipe-roof based on model of anisotropic plate on elastic foundation

To cite this article: WU Song *et al* 2020 *IOP Conf. Ser.: Earth Environ. Sci.* **570** 052070

View the [article online](#) for updates and enhancements.



EXTENDED ABSTRACT DEADLINE: DECEMBER 18, 2020

239th ECS Meeting
with the 18th International Meeting on Chemical Sensors (IMCS)

May 30-June 3, 2021

SUBMIT NOW →

The banner features a red top section with white text, a dark blue middle section with white text and logos, and a light blue bottom section with white text. The ECS logo is on the left, and the IMCS 18th logo is on the right. The background of the banner includes faint icons of a shopping cart, a person, and a yen symbol.

Analysis of mechanical behavior of a pipe-roof based on model of anisotropic plate on elastic foundation

WU Song¹, ZHI Cheng-jie¹, TANG Hua², DAI Yong-hao²

¹ Changjiang Institute of Survey, Planning, Design and Research Co., Ltd., Wuhan Hubei 430010, China

² State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan Hubei 430071, China

Abstract: According to the equivalent stiffness method of elastic theory, pipe roofs can be regarded as stiffening ribs and reinforced rock and soil can be regarded as an isotropic plate. Hence, pipe roofs and reinforced rock and soil in the local area of tunnel's arch crown can be considered as an anisotropic plate on an elastic foundation with specific constraints on the four edges. To improve the traditional elastic foundation beam model for pipe roofs, an anisotropic plate model of a pipe roof on an elastic foundation is established. The hyperbolic model is adopted for the foundation reaction. The mechanical model of a pipe roof is extended from one to two dimensions and solved by finite element method via the COMSOL partial differential equation (PDE) module. The model is used to analyze the mechanical behavior of a pipe-roof for a shallow-buried and soft rock highway tunnel. Meanwhile, the calculated results of the hyperbolic and traditional models (Pasternak model and Winkler model) are compared and analyzed. The results show that the following: (1) The stress and deformation of pipe roofs mainly appear in the section no more than 6 m ahead of the working face. The pipe roof plays a role of a supporting beam to improve the stabilization of the working face. It can effectively transfer the upper load to the un-excavated section and better distribute the stress of surrounding rock. (2) The 2D pipe-roof model overcomes defects of traditional models that cannot simulate the plane stress of the reinforced area and provides a reference for the design and construction of pipe-roofs. The longitudinal bending moment (M_y) is about 14 times the size of the lateral bending moment (M_x), which indicates that the pipe-roof has an obvious effect on M_y along the tunnel. M_x however cannot be neglected. (3) The maximum deflection difference between the hyperbolic model and Pasternak model is about 5%, and the calculated results are in good agreement with each other. The hyperbolic model considers the monotonous and bounded characteristics of sub-grade reaction varying with the displacement. The basis of the model is verified and extent of its accuracy is demonstrated.

1. Introduction

In recent years, pipe-roof methods have rapidly developed in tunnel and underground works ^[1]. As a major auxiliary construction method, the pipe-roof method is mainly used for tunnel construction under unfavorable geological conditions, such as weak, sandy gravel stratum or soft rock, rock piles, and broken zone stratum ^[2]. In addition, the pipe-roof method is widely used in the construction process of new tunnels passing through existing structures ^[3-4]. However, despite the pipe-roof method's popularity, its design is still mainly based on experience and engineering analogies, with a lack of relevant theoretical research.

From the results to date, research on the mechanical behavior of pipe-roofs can be divided into two



Content from this work may be used under the terms of the [Creative Commons Attribution 3.0 licence](https://creativecommons.org/licenses/by/3.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

aspects: (1) *numerical simulation*. By improving soil parameters to consider the role of pipe-roof reinforcement, the pipe-roof and surrounding rock are taken as the reinforced rock mass for numerical calculation [5]. While this method is simple to calculate, the increase in magnitude of soil parameters is arbitrary. (2) *Structural mechanics analysis*. The support stiffness is calculated using structural mechanics, mainly by including a simple beam model, simple beam model on elastic foundation [6], or Winkler elastic foundation beam model with rigid fixed end [7-8]. At present, theoretical research of pipe-roof pre-support is mainly based on the elastic foundation beam model, which has the characteristics of clear mechanical behavior and simple calculation, but still suffers some shortcomings. For example, in the traditional model, the unexcavated section in front of the tunnel face is simply considered as a foundation beam with constant coefficient of subgrade reaction. In fact, the reaction force of the foundation changes nonlinearly with increasing displacement, resulting in a section where the coefficient is decreased and uncertain mechanical behavior of the pipe-roof. In our study, based on the small deflection bending theory of anisotropic plates, the traditional 1D elastic foundation beam model of pipe-roofs is improved by considering the boundedness of the foundation reaction. A 2D anisotropic plate model on elastic foundation is established for simulating the mechanical behavior of the pipe-roof. The results of the hyperbolic foundation model are compared with those of traditional foundation models (Pasternak and Winkler), and the stress and deformation of pipe-roofs are discussed.

2. Pipe-roof model of anisotropic plate on elastic foundation

2.1. Discussion of foundation reaction calculation method

The two traditional calculation models of elastic foundation are the Winkler and Pasternak foundation models. The Winkler model uses a series of springs with fixed stiffness to simulate the foundation, where the reaction force and displacement have a positive linear relationship. The reaction force of the foundation is expressed as $p(x, y) = kw(x, y)$. The Pasternak model considers the shear interaction between spring elements and improves on the defect of discontinuous foundation deformation in the Winkler model. The expression of the foundation reaction force is as follows:

$$p(x, y) = kw(x, y) - G \left(\frac{\partial^2 w(x, y)}{\partial x^2} + \frac{\partial^2 w(x, y)}{\partial y^2} \right).$$

In the above formula, $p(x, y)$ is the foundation reaction force, $w(x, y)$ is the foundation displacement, and k and G are the soil elastic parameters.

In the above model, the foundation reaction force monotonically increases with displacement, but the foundation reaction force is not bounded; hence, the ultimate reaction force and displacement are not considered. In reality, the reaction force increases with displacement up to a peak value, leading even to possible foundation damage.

To date, many calculation models of passive earth pressure have considered displacement in soil mechanics, such as sine function [9], hyperbolic function [10], and exponential function [11]. The hyperbolic function is used to express the relationship between reaction force and displacement. In the range of limit equilibrium displacement, the foundation reaction monotonically and boundedly increases with displacement, as shown in Figure 1. The hyperbolic function is as follows:

$$p = \frac{w}{A + Bw}, 0 \leq w \leq w_u \quad (1)$$

In the formula, A and B are undetermined parameters, p is foundation reaction force, w is foundation displacement and w_u is foundation displacement in the limit state.

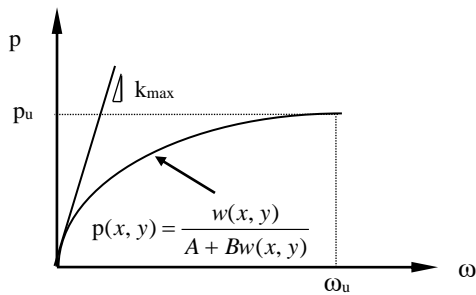


Figure 1. Hyperbola curve of subgrade reaction and displacement

Formula (1) can be rewritten as follows: $\frac{w}{p} = A + Bw$, when $w \rightarrow 0$, $A = \frac{1}{(p/w)}|_{w \rightarrow 0} = \frac{1}{k_{max}}$; k_{max} is the initial and maximum resistance coefficient. When $w = w_u$, $p = p_u$, $B = \frac{1}{p_u} - \frac{1}{k_{max}w_u}$ can be obtained, where p_u is the sub-grade reaction in the limit state. The parameters A and B in the nonlinear hyperbolic foundation model can be determined by indoor test or field plate load test. In particular, when $B = 0$, $p = k_{max}w$, this becomes the Winkler model.

2.2. Model establishment

Existing mechanical models of pipe-roofs mainly use a single steel pipe as the studied object, as is the case for Winkler and Pasternak beam on elastic foundation models. These methods have some defects, as they fail to consider the arch effect of the tunnel's cross section and the grouting reinforcement effect of the pipe-roof. As the ultimate bearing capacity of the surrounding rock foundation is ignored, the stress of the pipe-roof cannot be well simulated. In practice, the surrounding rock is strengthened by the pipe-roof grouting, and the loose surrounding rock above the arch roof is consolidated and dense, which produces plate and shell effects to a certain extent.

According to the method of stiffness equivalence [12] in elastic theory for stiffened plates, to change isotropic plates with stiffeners into anisotropic plates without ribs, the stiffness of ribs can be included in the plate stiffness to simplify calculation. During the grouting process, the pipe-roof can be regarded as a stiffening rib, the rock and soil reinforced by grouting can be regarded as an isotropic plate, and the combination of them can be regarded as an isotropic plate with stiffening ribs.

During tunnel excavation, the combined pipe-roof and reinforced rock and soil not only bear the overburden pressure of surrounding rock near the tunnel face but also the elastic resistance of rock and soil at the front of the tunnel face. Therefore, the pipe-roof and reinforced rock and soil in a partial region of the tunnel vault can be regarded as an anisotropic plate on an elastic foundation with specific constraints on four sides. The pipe-roof model of the anisotropic plate on an elastic foundation is shown in Figure 2.

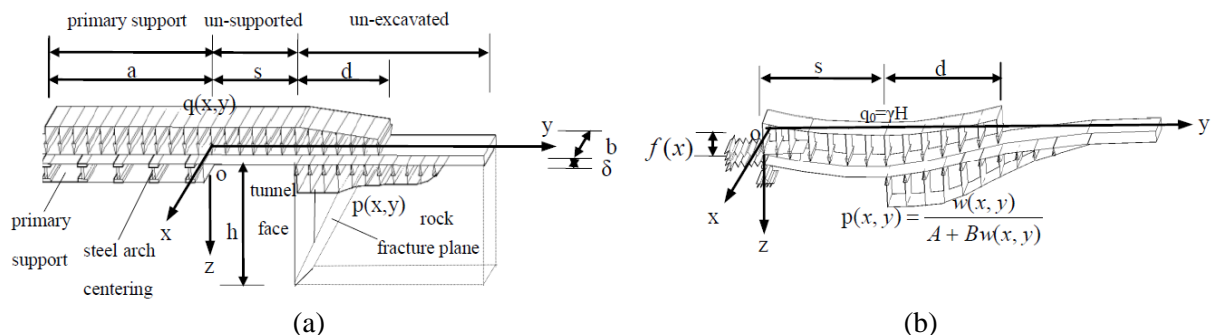


Figure 2. Mechanical model of pipe-roof during the excavation of working face

Figure 2(a) shows that the anisotropic plate is mainly divided into four parts: (1) The range $y < 0$,

where excavation and support has been completed; $y = 0$ position is regarded as the elastic fixed end with initial deflection. (2) The range $0 < y \leq s$, which has been excavated without support; the pipe-roof only bears the overburden pressure $q(x, y)$ of surrounding rock. (3) The range $s < y \leq s + d$, which is not excavated but relaxed; the pipe-roof bears both the surrounding rock pressure $q(x, y)$ and the reaction $p(x, y)$. (4) The range $s + d < y \leq L$, which is undisturbed; the pipe-roof only bears $p(x, y)$ caused by deformation. As shown in Figure 2 (b), h is the excavation height of the tunnel, $d = h \tan(45^\circ - \varphi/2)$, and φ is the internal friction angle of the surrounding rock.

According to elastic theory^[13], the fourth-order partial differential equation (PDE) for the bending problem of anisotropic plates on elastic foundation is as follows:

$$D_1 \frac{\partial^4 w}{\partial x^4} + D_2 \frac{\partial^4 w}{\partial y^4} + 2D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} = q - p \quad (2)$$

where w is the plate deflection; D_1 , D_2 , and D_3 are stiffness coefficients; q is the transverse load per unit area; and p is the sub-grade reaction per unit area. Substituting Formula (1) into (2):

$$D_1 \frac{\partial^4 w}{\partial x^4} + D_2 \frac{\partial^4 w}{\partial y^4} + 2D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{w}{A + Bw} = q, (0 \leq w \leq w_u) \quad (3)$$

To simplify the calculation, an anisotropic plate with a pipe-roof can be regarded as an anisotropic plate with single direction stiffeners. Assuming that the pipe-roof is arranged along the y -axis direction, the principal stiffness^[14] of the anisotropic plate is

$$D_1 = D_3 = \frac{E\delta^3}{12(1-\mu^2)}, \quad D_2 = \frac{E\delta^3}{12(1-\mu^2)} + \frac{E'I_2}{a_2}$$

where δ is the thickness of the plate, E and μ are the modulus of elasticity and Poisson's ratio of the plate, respectively, E' is the modulus of elasticity of the pipe-roof, I_2 is the section moment of inertia for the pipe-roof, and a_2 is the transverse spacing of the pipe-roof.

The internal force of an anisotropic plate can be expressed by the deflection function as follows:

$$M_y = -D_1 \left(\frac{\partial^2 \omega}{\partial x^2} + \mu \frac{\partial^2 \omega}{\partial y^2} \right), M_x = -D_2 \left(\mu \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right), M_{xy} = -2D_3 \frac{\partial^2 \omega}{\partial x \partial y} \quad (4)$$

When the tunnel excavation face is close to the front end of the pipe-roof, the pipe-roof and the reinforced rock and soil can be regarded as a plate on an elastic foundation with finite length. It is assumed that $y = 0$ has an initial displacement and initial rotation angle, $y = L$ has a displacement and rotation constraint, and $x = \pm b/2$ has no rotation angle. The boundary conditions are as follows:

$$y = 0, w(x, y)_{y=0} = f(x), \frac{\partial w}{\partial x} \Big|_{y=0} = f'(x), \frac{\partial^2 w}{\partial y^2} \Big|_{y=0} = 0; y = L, w(x, y)_{y=L} = 0, \frac{\partial w}{\partial x} \Big|_{y=L} = 0, \frac{\partial w}{\partial y} \Big|_{y=L} = 0; x = \pm b/2, \frac{\partial w}{\partial x} \Big|_{x=\pm b/2} = 0.$$

where $f(x)$ is the distribution function of the initial deflection of the elastic fixed end at $y = 0$, and $f'(x)$ is the first derivative. On the one hand, due to the transverse displacement distribution law of pipe-roof and reinforced rock and soil being similar to that of the tunnel surface, the peck method can be used to

fit the initial deflection $f(x)$. The displacement prediction model is $s = s_{\max} \exp\left(-\frac{x^2}{2i^2}\right)$, where the

coefficient i can be found by fitting the surface settlement monitoring data. On the other hand, due to the settlement of the tunnel vault being close to that of the pipe-roof, the peak deflection s_{\max} of the pipe-roof can be simulated by monitoring the value of the tunnel vault. If there is no field monitoring data, it can also be obtained by numerical simulation.

The above boundary conditions are suitable for steel flower tube consolidation grouting and soft ground with good grouting effect. For the ground with holeless steel pipes and poor grouting effect,

the lateral load-bearing arch effect is weak; hence, the above boundary is not suitable.

2.3. Small deflection bending of thin plate solved by COMSOL PDE

COMSOL Multiphysics is finite element numerical simulation software. Its mathematical module has many branches, among which the PDE module can solve three kinds of PDEs: coefficient, generalized and weak solution types. According to the needs of research problems, control equations and boundary conditions can be set to solve small deflection bending of anisotropic plates.

The deflection equation for the anisotropic plate on elastic foundation is of the fourth order, but COMSOL can only solve up to the second order. By introducing the intermediate variables $P = w_{xx}$, $Q = w_{yy}$, the fourth order PDE is reduced to a second order PDE and solved; that is

$$\left. \begin{aligned} D_1 * P_{xx} + D_2 * P_{yy} + 2 * D_3 * Q_{yy} &= q - k * w + G_x * P + G_y * Q \\ P &= w_{xx} \\ Q &= w_{yy} \end{aligned} \right\} \quad (5)$$

In the COMSOL PDE module, the generalized PDE is used for the calculation. The basic equation is as follows:

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot \Gamma = f, u = [w, P, Q]^T, \nabla = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right] \quad (6)$$

In the above formula,

$$\Gamma = \begin{bmatrix} D_1 * P_{xx} & D_2 * P_{yy} + D_3 * Q_{yy} \\ w_{xx} & 0 \\ 0 & w_{xx} \end{bmatrix}, f = \begin{bmatrix} q - k * w + G_x * P + G_y * Q \\ P \\ Q \end{bmatrix}, e_a = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, d_a = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

When using COMSOL PDE to calculate the deflection of anisotropic plate, the following steps are taken. First, establish the geometric model according to the plate size. Then, apply boundary conditions around the plate based on the PDE coefficient. Finally, solve the equation group (5) to obtain the deflection distribution of the plate to obtain the internal and reaction forces of the plate.

3. Examples and discussion

3.1. Examples

According to the actual conditions of the Diantou tunnel along the Yunnan Da-yong highway (surrounding rock level: V), the tunnel parameters are recorded as follows: tunnel burial depth $H = 12$ m, excavation height of tunnel upper bench $h = 5$ m, and single footage $s = 1.2$ m. In the single excavation process, $s_{max} = 0.01$ m is obtained from monitoring data of the vault settlement, and $i = 14.43$ m is obtained by fitting monitoring data of the surface settlement according to the peck formula. Hence, the initial deflection at $y = 0$ is $\omega_0 = 0.01 \exp(-0.0024x^2)$ (units: m). The calculation range is shown in Figure 2. The parameters of the anisotropic plate on elastic foundation pipe-roof model are as follows: spacing between the pipe-roof $b_0 = 0.4$ m, width of the plate $b = 3b_0$, and plate thickness of the $\Phi 108$ pipe-roof $\delta = 108$ mm. Through an equivalent calculation, the elastic modulus of the plate $E = 3.698$ GPa and Poisson's ratio $\mu = 0.3$. The parameters of the surrounding rock are the following: volume weight $\gamma = 20$ kN/m³, internal friction angle $\phi = 20^\circ$, and transverse uniform load $q = 240$ kPa.

To obtain unknown parameters A and B of foundation reaction force, a field plate load test is performed, as shown in Figure 3. Multiple load tests were conducted, and the curve of load p and displacement w are shown in Figure 4. According to the hyperbolic function fitted by the measured data, $A = 1/100$ and $B = 1/5000$ can be obtained by combining the ultimate displacement $w_u = 25$ mm, ultimate subgrade reaction $p_u = 1667$ kPa, and maximum slope value k_{max} of the load and displacement

near $w = 0$. Substitution of A and B into Formula (1) affords $p = \frac{w}{\frac{1}{100} + \frac{w}{5000}}, 0 \leq w \leq 25\text{mm}$.



Figure 3. Field plate load test

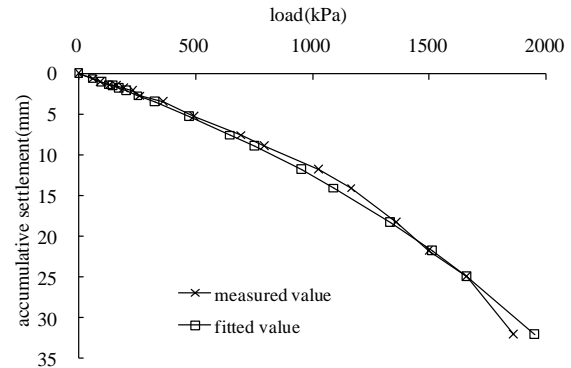
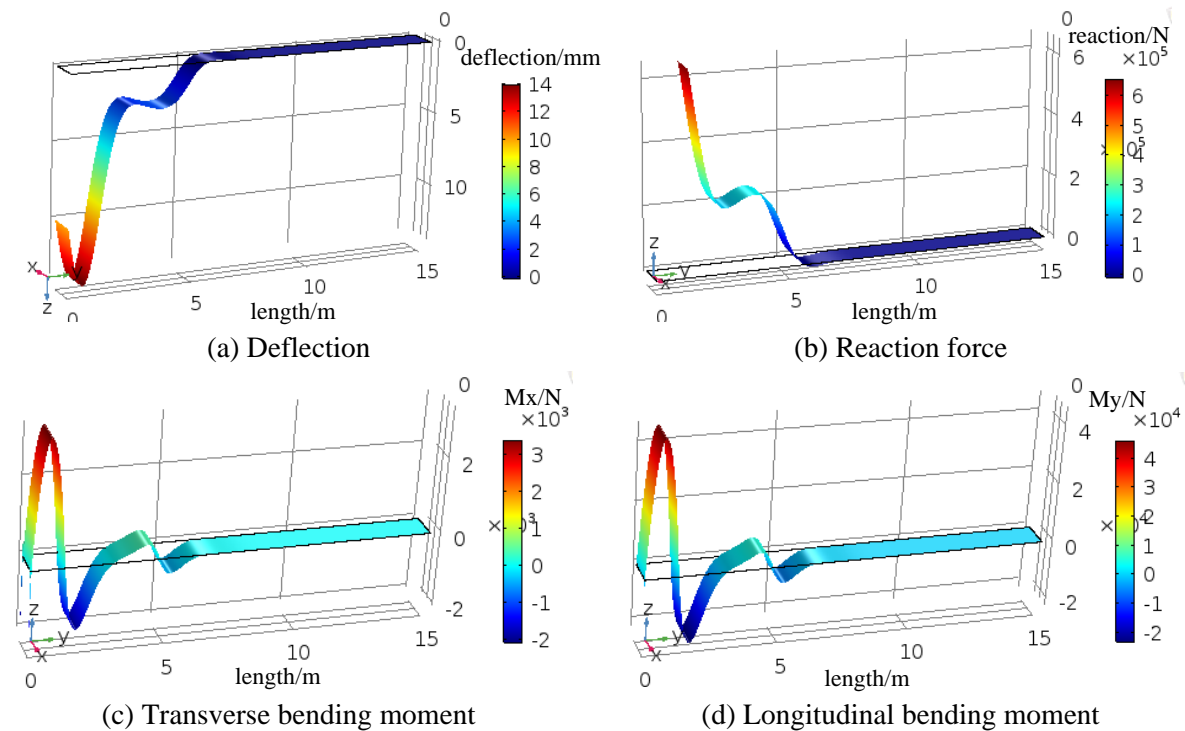
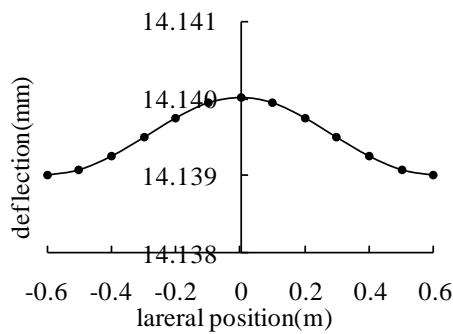


Figure 4. Curve of load and displacement

The deflection and internal force distribution of the anisotropic plate on elastic foundation $\Phi 108$ pipe-roof can be obtained through the COMSOL PDE module solution and the calculation method of the nonlinear hyperbolic foundation model, as shown in Figure 5. The distribution of transverse and longitudinal bending moments along the axial center line of the anisotropic plate on elastic foundation of pipe-roof is shown in Figure 6.





(e) Transverse distribution of the deflection
Figure 5. Curve of pipe-roof stress and deformation

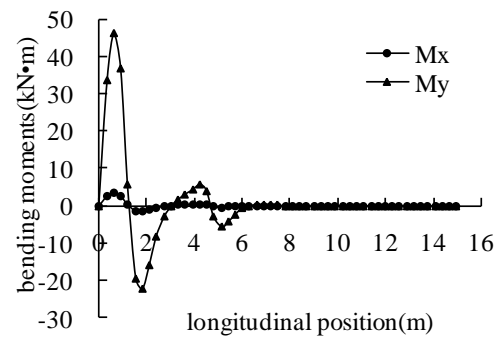


Figure 6. Comparison of transverse and axial bending moments

Figure 5(a) shows that during the tunnel excavation, the deflection of the anisotropic plate on elastic foundation pipe-roof is in the shape of a groove along the y-axis. The maximum plate deflection of the $\Phi 108$ pipe-roof model is 14.13 mm, occurring near the excavation face center $x = 0$ and $y = 0.6$. Combining Figure 5(e), the anisotropic plate on elastic foundation pipe-roof model can simulate the transverse displacement distribution of the tunnel vault with the deflection changing along the x-axis. It is assumed that the model of pipe-roof at $y = 0$ is normally distributed along the x-axis. With the excavation of the tunnel face, the peak deflection will increase and move forward along the excavation direction.

Figure 5(b) shows that in the range of $y > 6$ m, the foundation reaction is basically zero, while the rock pressure is less than the foundation reaction within 0–4 m around the excavation face, indicating that the pipe-roof plays a role of a supporting beam. The pipe-roof can transfer the upper load near the excavation face to the un-excavated section and better distribute the rock stress.

Figure 5(c) and (d) shows that the stress of pipe-roof corresponds to deflection. The maximum lateral (M_x) and longitudinal (M_y) bending moments of the pipe-roof model are 46187 and 3395 N·m, occurring near the tunnel face. The bending moment is mainly concentrated within 6 m of the tunnel face, while the bending moment of 6 m outer plate approaches to zero, which indicates that the longitudinal influence of tunnel excavation is up to about 6 m, close to one times the bench height. Therefore, for soft surrounding rock and other unfavorable geological sections, the multi-bench or circular excavation with reserved core soil methods should be preferred to reduce excavation height to decrease the longitudinal influence and improve the face stability. Combined with Figure 6, since the anisotropic plate model on elastic foundation pipe-roof is planar, comparative analysis shows the bending moment; M_y is about 14 times the size of M_x , which indicates that the pipe-roof has an obvious effect on M_y along the tunnel. M_x however cannot be neglected.

3.2 Discussion

According to the three foundation calculation methods described in Section 2.1, the calculated deformation and stress comparison results of the pipe-roof are shown in Figure 7. While there are some differences in calculation results between the three models, the overall behavior is the same. On one hand, the deflection order of the three models is Hyperbolic model > Pasternak model > Winkler model. The maximum deflection difference between the Pasternak and Winkler models is about 6%, which is consistent with the conclusion of Jia et al. [14], while the difference between the Hyperbolic and Pasternak models is about 5%. This shows that the Hyperbolic model is in good agreement with traditional models, which verifies the basis of the improved foundation model. On the other hand, near the tunnel face, the order of maximum reaction force is Pasternak model > Winkler model > Hyperbolic model. In the range of 3 m from the tunnel face, the Hyperbolic model is the maximum. The Hyperbolic model considers the monotonous and bounded characteristics of sub-grade reaction varying with the displacement, overcoming the shortcomings of traditional models and demonstrating better accuracy to an extent.

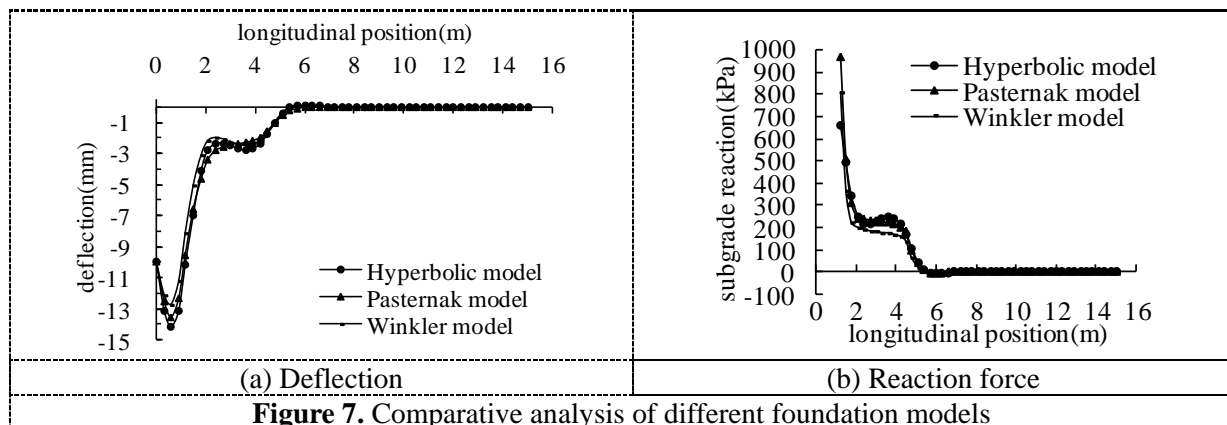


Figure 7. Comparative analysis of different foundation models

4. Conclusion

(1) The anisotropic plate on elastic foundation pipe-roof model is proposed based on traditional beam models on an elastic foundation. The mechanical model of the pipe-roof is extended from one to two dimensions, overcoming the shortcomings of traditional models that cannot simulate the plane stress of the reinforced area from the pipe-roof or consider the boundedness of the sub-grade reaction. The longitudinal and lateral forces and deformations of the pipe-roof and reinforced area can be calculated, providing a reference for the design and construction of pipe-roofs.

(2) The model is solved by COMSOL PDE and used to analyze the mechanical behavior of the pipe roof for shallow-buried and soft rock highway tunnel. During tunnel excavation, the longitudinal influence of the pipe-roof is no more than 6 m ahead of the working face, while M_y is about 14 times the size of M_x , indicating that the pipe-roof has an obvious effect on M_y along the tunnel. M_x cannot however be neglected.

(3) The maximum deflection difference between the Hyperbolic and Pasternak models is about 5%, and the calculated results are in good agreement with each other. The Hyperbolic model considers the monotonous and bounded characteristics of sub-grade reaction varying with the displacement. Thus, the basis of the model is verified and extent of its accuracy is demonstrated.

References:

- [1] Wang Mengshu. Underground Engineering Shallow Boring Technology [M]. An Micro Education Press, 2004.12.
- [2] Zhao Jianping. Shallow burial tunnel shed pipe shed support mechanism and effectiveness [D]. Changsha: Central South University, 2005.
- [3] Guan Baoshu. Introduction to Tunnel Mechanics [M]. Chengdu: Southwest Jiao Tong University Press, 1993, 3.
- [4] Zhou Shunhua. Principles of pipe roof applied to shallow-buried tunnels in soft ground[J]. Chinese Journal of Rock Mechanics and Engineering, 2005, 24(14): 2 565–2 570.
- [5] Li Chao. Mechanism of pipe-roof in shallow-buried tunnels and 3D numerical simulation of construction process[D]. Tianjin: Tianjin University, 2010.
- [6] CHEN Hao, JIANG Jingshan, YAO Haibo. Construction and distortion analysis of pipe-roof installed in Chongwenmen metro station underneath existing metro line[J]. Tunnel Construction, 2006, 26(1): 78–80.
- [7] LI Jian, TAN Zhongsheng, YU Yu, et al. Analysis of deformation monitoring and mechanical behaviors of big pipe-roof for shallow-buried large-span tunnel to underpass highway[J]. Chinese Journal of Rock Mechanics and Engineering, 2011, 30(Supp.1): 3 002–3 008.
- [8] GOU Deming, YANG Junsheng, ZHANG Ge. Deformation monitoring and mechanical behaviors of pipe-roof in shallow tunnels[J]. Chinese Journal of Rock Mechanics and Engineering, 2007, 26(6): 1 258–1 264.
- [9] XU Ri-qing. Methods of earth pressure calculation for excavation[J]. Journal of Zhejiang University (Engineering Science), 2000, 34 (4): 370-375.

- [10] LU Kun-lin, YANG Yang. Calculation of Unlimited Passive Earth Pressure[J]. Journal of Hefei University of Technology: Natural Science, 2009, 32(4): 523-527.
- [11] MEI Guo-xiong, ZAI Jin-min. Earth Pressure Calculating Method Considering Displacement[J]. Rock and Soil Mechanics, 2001, 22(4): 83-85.
- [12] Xu Zhilun. Elasticity mechanics[M]. 5th ed. Beijing: Higher Education Press, 2016.
- [13] Yang Duansheng, Huang Yan. Bending of orthotropic rectangular plated on double-parametric elastic foundation[J]. Engineering Mechanics, 2009, 26(3): 26-30.
- [14] JIA Jinqing, WANG Haitao, TU Bingxion, et al. Analytical approach and field monitoring for mechanical behaviors of pipe roof reinforcement[J]. Rock and Soil Mechanics, 2010, 31(6): 1858-1864.

Acknowledgments

This work was supported by grants from the National Key Research and Development Program of China (2016YFC0401804), the Traffic Science and Technology Project of Yunnan Province [(2016)140(A)], and the Independent Innovation Project of Changjiang Design Institute (CX2018Z08).