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Identification of non-linear stress–strain–time relationship of soils using genetic algorithm

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SUMMARY

Recognition of non-linear constitutive rock/soil model from experimental results is often multi-modal in the large parameter space. A genetic evolution algorithm is thus proposed for its recognition, including that of structure of the model and coefficients in the model. The structure of the model can be firstly determined according to mechanical mechanism if the mechanism is clearly understood or searched by using evolutionary algorithm. The coefficients to be determined are then searched in global optional space. With the new evolutionary algorithm, the non-linear stress–strain–time constitutive law to describe strain softening behaviours of diatomaceous soil under consolidated and undrained state was recognized by learning stress–strain–time behaviour of an intact sample under consolidated pressure of $\sigma_c = 0.1$ MPa and strain velocity of $\dot{\varepsilon}_a = 0.175\%/\text{min}$. This model gave reasonable prediction for diatomaceous soils under varying consolidated pressures (0.1–3.5 MPa) and strain velocities (0.0044–1.75%/min). It indicates that the methodology proposed in this paper is robust enough and strongly attractive for recognition of non-linear constitutive model of soil and rock materials. Copyright © 2002 John Wiley & Sons, Ltd.

1. INTRODUCTION

Recognition of non-linear behaviours of rocks and soils becomes increasingly interesting and is one of the key important problems in design, stability analysis, prediction and control of failure for geotechnical engineering projects. Many researchers gave their effort to build these non-linear constitutive models based either on mechanics [1–6] or on back recognition from experimental data [7]. The former needs to understand well the mechanism of deformation and failure of soils. The mechanical mechanism determines the type of the structure of the model. The latter is an inverse problem. It has to determine the unknown structure and parameters of model. Thus, it has the large parameter space and is highly multimodal. There are three main types of search methods for this recognition: calculus-based, enumerative, and random. Since recognition of highly non-linear constitutive material model is of the large parameter space and

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highly multimodal, calculus-based and enumerative techniques were discounted as being either not robust enough or not efficient enough to handle this problem. Thus, the two most efficient algorithms are the genetic algorithm and the simulated annealing technique.

In this paper, mechanical methods and intelligent back-recognition are integrated. Structure of the constitutive model is determined from its mechanical mechanism. The genetic algorithm [8] was then chosen for its biological and evolutionary appeal to find the set of unknown parameters that best matched modelling prediction with experimental data. The details of this genetic algorithm technique and the implementation for non-linear stress–strain–time relationship of a soil are the subject of this paper.

2. GENETIC ALGORITHM

A genetic algorithm operates on the Darwinian principle of 'survival of the fittest'. An initial population of size w is created from a random selection of the parameters in the parameter space. Each parameter set represents the individual's chromosomes. Each individual is assigned a fitness based on how well each individual's chromosomes allow it to perform in its environment. Through selection, crossover, and mutation operations, with the probabilities P_s, P_c, and P_m, respectively, the next generation is created. Fit individuals are selected for mating, whereas weak individuals die off. Mated parents create a child with a chromosome set that is some mix of the parent's chromosomes. For example, parent 1 has chromosomes HIJKL, whereas parent 2 has chromosomes ABCDE, one possible chromosome for the child is HICDE, where the position between the chromosomes I and C is the crossover point. There is a small probability that one or more of the child's chromosome will be mutated, e.g. the child ends up with chromosome HOCDE. The process of mating and child creation is continued until an entire population of size w is generated, with the hope that strong parents will create a fitter generation of children; in practice, the average fitness of the population tends to increase with each generation. The fitness of each of the children is determined, and the process of selection/crossover/mutation is repeated. Successive generations are created until very fit individuals are obtained.

3. EXPERIMENTAL DATA FOR RECOGNITION OF MODEL

The triaxial tests were conducted on diatomaceous mudstones specimens consisting of residual diatomaceous, clay, and pozzolana. The cylindrical specimens with diameter 5 cm and height 10 cm were drilled from cubic of $40 \times 40 \times 40\ \text{cm}^3$. The mudstones are specific gravity of $2.183\ \text{g/cm}^3$, natural water content w_n of 119.6%, liquid limit w_l of 172.7%, plastic limit w_p of 94.7%, and compressive index c_c of 1.458. The specimens were completely saturated. It is a high porosity and high compressive material. Two types of specimens, intact and precut (see Figure 1), were used to investigate effect of discontinuities. The consolidated and undrained triaxial tests were conducted with various strain velocity $\dot{\varepsilon}_a$ of 0.0044–1.75%/min. Its standard value is 0.175%/min. In order to investigate time-dependent behaviours, the creep tests were also conducted.

It can be seen from Figures 2 and 3 that it exists a low strain softening and deviator stress decreased gradually after peak strength if previous consolidation pressure is larger than

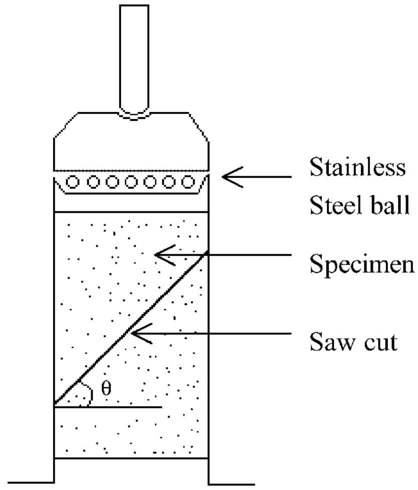


Figure 1. Schematic view of the precut specimen.

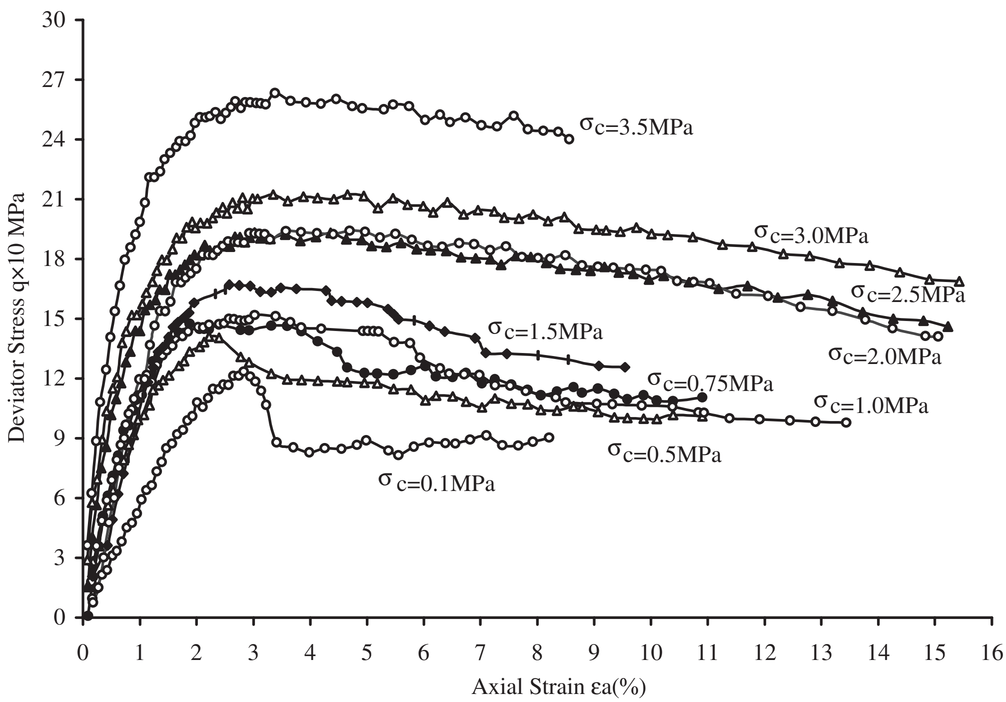


Figure 2. The measured data for intact diatomaceous soil specimens.

1.5 MPa. This belongs to normal consolidation state. Contrastively, strain softening abruptly occurs after peak strength and then deviator stress decrease to a value, residual strength, as over consolidation state. Therefore, time dependency is important to strain softening of soils (Figure 4). There raises a problem: How to determine accurately a constitutive law, a non-linear

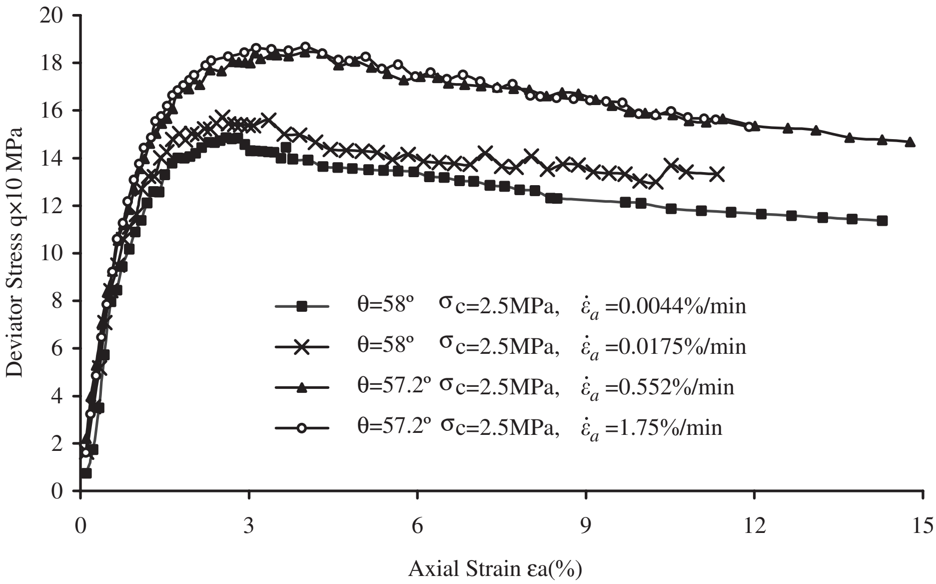


Figure 3. The measured data for diatomaceous soil specimens with a joint at different angle θ.

stress–strain–time relationship, for these complicated behaviours? A recognition based on genetic algorithm shown as follows can be constructed for this.

4. METHODOLOGY FOR IDENTIFYING NON-LINEAR STRESS–STRAIN–TIME RELATIONSHIP OF SOILS

Identification of non-linear stress–strain–time relationship of soil starts with determination of the underlying response mechanism. It is used to determine expression, i.e. the structure, of the model.

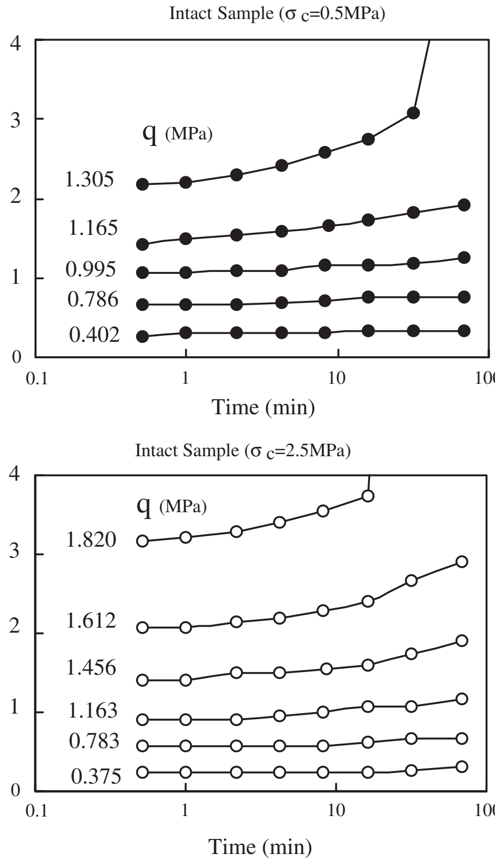
And then coefficients in the tentative model are recognized by using evolutionary algorithm.

4.1. Basic stress–strain relationship

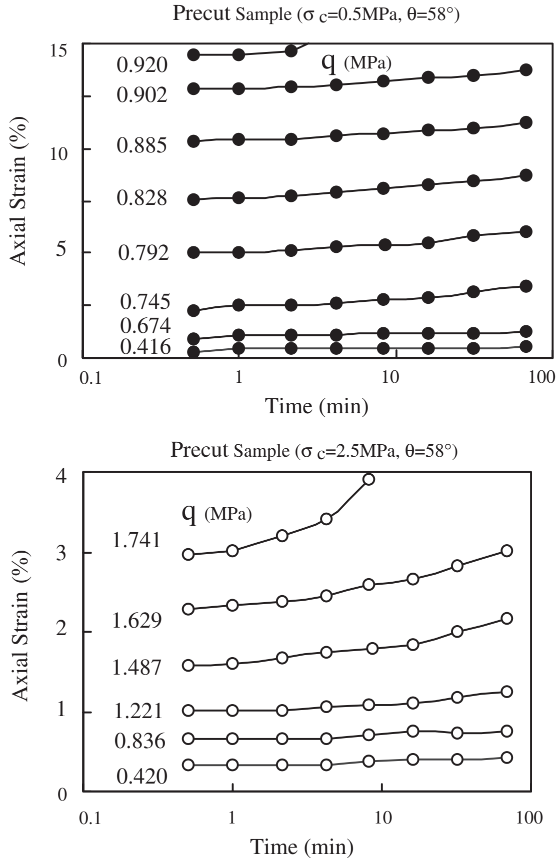
The incremental stress–strain relation for geotechnical materials is basically written by [1,9,10]

$$\begin{Bmatrix} \mathrm{d}v \\ \mathrm{d}\varepsilon \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{Bmatrix} \mathrm{d}p' \\ \mathrm{d}q \end{Bmatrix} \tag{1}$$

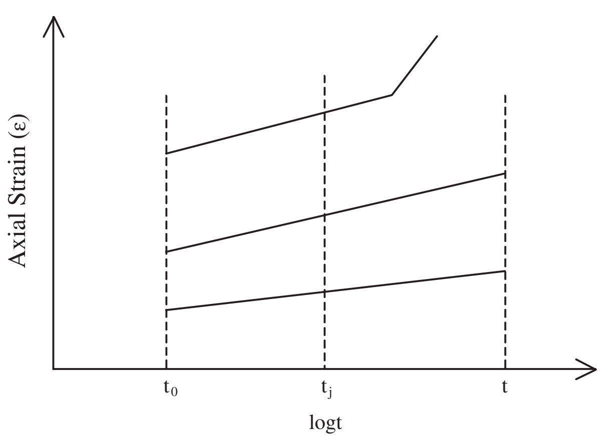
Figure 4. Relationship between strain and time stress control under consolidated and undrained condition: (a) the experimental results for diatomaceous soil without joint; (b) the experimental results for diatomaceous soil with a joint at given angle; and (c) a representative ε–log t plot.


Intact Sample (σ c=0.5MPa)
q (MPa)
1.305
1.165
0.995
0.786
0.402
Time (min)
Intact Sample (σ c=2.5MPa)
q (MPa)
1.820
1.612
1.456
1.163
0.783
0.375
Time (min)

(a)


Precut Sample (σ c=0.5MPa, θ=58°)
q (MPa)
0.920
0.902
0.885
0.828
0.792
0.745
0.674
0.416
Axial Strain (%)
Time (min)
Precut Sample (σ c=2.5MPa, θ=58°)
q (MPa)
1.741
1.629
1.487
1.221
0.836
0.420
Axial Strain (%)
Time (min)

(b)


Axial Strain (ε)
t0
tj
t
logt

(c)

where dv is volume strain increment, $d\varepsilon$ is shear strain increment, dp' is effective stress increment, dq is deviator stress increment. According to yield function and correlation flow rule, the coefficients C_{11}, C_{12}, C_{21}, and C_{22} in Equation (1) can be represented by

$$C_{11} = \left[1 + \ln\left(\frac{p'}{p'_0}\right)\right] + \frac{1}{K}, \qquad C_{12} = \frac{\chi}{Mp'}$$

$$C_{21} = C_{12}, \qquad C_{22} = \frac{\chi}{M^2 p'[1 + \ln(p'/p'_0)]} + \frac{1}{3G} \tag{2}$$

where

$$\chi = \frac{\lambda - k}{1 + e_0}, \qquad M = \frac{6 \sin \varphi'}{3 - \sin \varphi'}$$

φ' is the effective friction angle, M is the slope of critical state line at surface p' and q, e_0 is initial pore ratio, λ is compressive coefficient, k is dilation coefficient, p' is average effective stress, p'_0 is average effective stress after consolidation, K is volume deformation modulus and G is shear modulus.

4.2. Stress–strain–time relationship

Since mechanical behaviours of diatomaceous soils investigated here is time dependent and similar to that of over-consolidated clay, the stress–strain–time constitutive model for it can be built by expanding Equation (1) through adding C_{13} and C_{23} items related to time as

$$\begin{Bmatrix} dv \\ d\varepsilon \end{Bmatrix} = \begin{bmatrix} C'_{11} & C'_{12} & C_{13} \\ C'_{21} & C'_{22} & C_{23} \end{bmatrix} \begin{Bmatrix} dp' \\ dq \\ dt \end{Bmatrix} \tag{3}$$

For the consolidated and undrained triaxial tests of soils, volume strain increment is zero [9,10], i.e. $dv = 0$. Therefore, Equation (3) can be simplified as

$$\{d\varepsilon\} = [C'_{21} \quad C'_{22} \quad C_{23}] \begin{Bmatrix} dp' \\ dq \\ dt \end{Bmatrix} \tag{4}$$

where

$$C'_{21} = m \frac{\chi'}{Mp' \times 10^{-1}},$$

$$C'_{22} = f \frac{\chi'}{M^2 p' \times 10^{-1}[1 + \ln(p'/p'_0)]}, \qquad C_{23} = \beta \frac{d\varepsilon}{t}, \quad \chi' = \frac{\lambda - k}{1 + ce_0}, \tag{5}$$

where m, f, β and c are coefficients to be determined.

According to the relationship between strain and time obtained in consolidated and undrained creep test (Figure 4), the following representative equation can be established by:

$$\varepsilon = a \log t + b \tag{6}$$

where a, b are coefficients to be determined.

Combining Equations (4), (5), and (6), the following equation can be obtained as:

$$d\varepsilon = \frac{a}{4.605\beta}\left[1 + \sqrt{1 - \frac{9.21}{a}(C'_{21}\,dp' + C'_{22}\,dq)}\right] \quad (7)$$

Obviously, Equation (7) is a representation of non-linear stress–strain–time law for diatomaceous soils. The problem is to determine unknown coefficients in Equation (7). Therefore, recognition of non-linear stress–strain–time law for diatomaceous soils reduces here to that of parameters. A good candidate evolutionary algorithm for this was genetic algorithm. After different tentative settings (different set of the coefficients $a,\ m,\ f,\ \beta$ and c in Equations (5) and (7)), our choice is to find a best set of coefficient set with minimum error (fitness) from experimental results. The recognition starts creation of initial tentative sets of coefficients and finishes through evolution of sets.

4.3. *Creation of initial tentative non-linear stress–strain–time relationship*

Initially, the w groups of tentative coefficient set can be randomly generated to generate w tentative non-linear stress–strain–time relationships represented by Equation (7). Since Knuth's subtractive algorithm is regarded as one of the best random number generators [8], random numbers may be generated using Knuth's method. The question is what is an appropriate population size w, number of the tentative models, must be addressed before any genetic algorithm calculations can be run. From Reference [11], appropriate population size is

$$w = O(d\chi^q) = O[(l/q)\chi^q]$$

where $d = l/q$, χ is the cardinality of chromosomes, i.e. for binary coding $\chi = 2, q$ is the size of the schema of interest, and l is the length of the chromosome string.

Among all search spaces, a special attention is paid to those that can be mapped onto the Euclidean vector space R^n, for some integer n, e.g. the set of polynomials of a given degree.

4.4. *Evaluation of tentative non-linear stress–strain–time relationship*

The quality of a tentative model can be easily derived by comparing, under given experimental conditions, the actual experimental responses with their numerical simulations obtained using that tentative set: a good model should give simulated results close to the experimental ones. A model with the tentative coefficient set is evaluated through the difference between the observed strain $\Delta\varepsilon_{mea}$ and the strain $\Delta\varepsilon_{com}$ computed from that model according to the experimental loading history:

$$\text{Fitness} = \frac{1}{N}\sqrt{\sum_{i=1}^{N}(\Delta\varepsilon_{com} - \Delta\varepsilon_{mea})^2} \quad (8)$$

Two errors were considered: the overall average error of all of the members of the population and the best individual with the smallest error. The average error is a measure of how well the population as a whole is doing, as well as how fast it is converging to the optimal solution. The best error simply indicates how well the genetic algorithm has done in finding a minimum cost solution.

If error represented by Equation (8) is not accepted or desired, then the tentative material models should be recreated by evolving. This evolution is carried out by reproduction, crossover and mutation operation performed on binary string representing constitutive model.

4.5. Evolution of tentative non-linear stress–strain–time relationship

The w groups of tentative non-linear stress–strain–time relationship are than evolved through reproduction, crossover and mutation on chromosomes. Reproduction is a process in which individual strings are copied according to their error function values, Fitness. Intuitively, we can think of the function *Fitness* as some measure of profit, utility, or goodness that we want to optimize. Coping strings according to their fitness values means that strings with a lower value have a higher probability of contributing one or more offspring in the next generation. This operator, of course, is an artificial version of natural selection, a Darwinan survival of the fittest among string creatures. This is the standard roulette wheel reproduction operator, with Monte Carlo selection with probabilities based on fitness levels. As no explicit local optimizing operator is used, we introduced fitness scaling; a procedure that enhances differences among similar fitness values, to improve fine tuning of the solutions. An elitist strategy is also implemented, preserving the best individual of the last iteration in the new population.

During crossover with binary coding, the crossover point may occur in the middle of one of the parameter string; this allows the child to have a parameter string that is a mix of the parent parameter strings and, consequently, the child may have an allel (parameter value) between the two alleles of the parents. Single-point crossover strategy is used here. The chromosome set of the first parent is mapped into the child. A crossover point is randomly chosen where the chromosome set of the second parent, overwrites the chromosome set of the parent, e.g., one possible chromosome set for the child. If it has probability P_c, then there is a $1 - P_c$ probability that the child would retain the entire chromosome set of the first parent. The following is an example for evolution of models to next generation

As before, the new created tentative constitutive models is tested using both the learning cases and the testing cases again. Their applicability is evaluated using Equation (8). The algorithm stops whenever the fitness of the best individual (error in the time–strain–stress space) becomes lower than the heuristically computed unavoidable error, or after a given number of generations.

4.6. Algorithm for genetic evolution of material constitutive models

Summarily, algorithm for genetic evolution of material constitutive models is described as follows.

Step 1: Collect a set of experimental data obtained in rock mechanical tests. The data set is divided into two groups. One is used as fitness cases to obtain constitutive model. Another is used as testing cases to appraise applicability of the learned constitutive model. Follow the following learning process to find the best model whose predictions both for learning cases and test cases agree well with the measurements.

Step 2: Generate randomly w groups of tentative constitutive models $\{w\} = \{w_1, w_2, \ldots, w_m\}$ as initial generation of model evolution.

Step 3: Errors are calculated using Equation (8) to evaluate fitness of each tentative model individual.

Step 4: If the calculated error is less than the allowable error, then genetic evolutionary procedure is finished. Otherwise, go to Step 5.

Step 5: Perform genetic operations on the binary (or real number) string presenting the current tentative model individuals to obtain w groups of new material model individuals. Then go to Step 3.

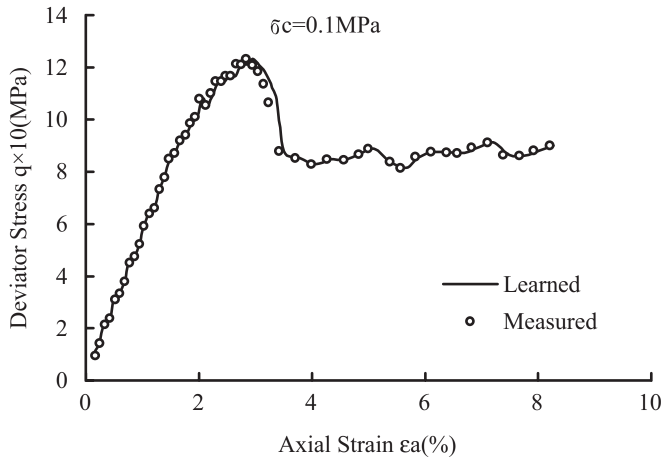


Figure 5. Learned results of constitutive model for the specimen no. 17 in Table I with $\dot{\varepsilon}_a = 0.175\%/\text{min}$, the data was used for building the constitutive model.

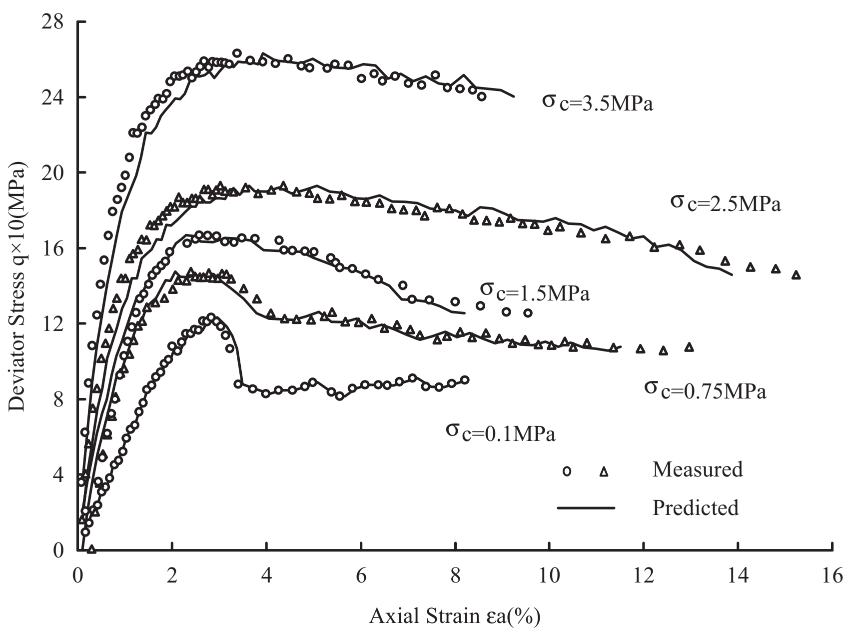


Figure 6. Comparison of generalization predictions of the recognized constitutive model and measurements for intact specimens with $\dot{\varepsilon}_a = 0.175\%/\text{min}$, the data was not used for building the constitutive model.

5. THE RECOGNITION OF NON-LINEAR STRESS–STRAIN–TIME RELATIONSHIP OF DIATOMACEOUS SOIL

Since mechanical behaviour at pre-peak strength of soils is often different from that at post-peak strength, the coefficients a, m, f, β and c in Equations (5) and (7) for these two states are separately determined. The critical issue is the choice of the search space in which to look for a solution. It should be large enough to include high-quality solutions, but not too large, as the search would then be intractable. The search space is thus determined to be $0 \leqslant a \leqslant 10, 0 \leqslant m, f, \beta \leqslant 100, 0 \leqslant c \leqslant 10$. The population size was determined to be $w = 1000$. The random seed number was set to be $S = 2000$. The specimen no. 17 in Table I, over consolidated with the pressure $\sigma_c = 0.1$ MPa and without precut, is only used as fitness case to obtain the non-linear stress–strain–time relationship of diatomaceous soil. The evolutionary parameters were

Table I. Parameters and cases used to determine the strain–stress–time constitutive model.*

Basic experimental parameters		$\lambda = 0.642,\ k = 0.071,\ v = 0.397,\ M = 1.75$				
Specimen no.	Consolidation pressure σ_c (MPa)	e_0	Strain velocity $\dot{\varepsilon}_a$ (%/min)	W(%)	Precut angle (deg)	ρ (t/m^3)
1†	0.5	2.75	1.75	128.5		1.324
2	0.5	2.75	0.7	129.64		1.331
3	0.5	2.75	0.35	14.605		1.307
4	0.5	2.75	0.0875	13.619		1.282
5	0.5	2.75	0.044	13.154		1.303
6	0.5	2.75	0.0175	124.57		1.339
7	0.5	2.75	0.0044			1.377
8	2.5	2.38	0.0044	108.77		1.323
9	2.0	2.54	0.0175	108.77		1.362
10	2.0	2.54	0.0583	104.98		1.357
11	2.0	2.54	1.75	122.11		1.355
12	2.5	2.76	0.175	123.15		1.306
13	0.5	2.75	0.175	124.31		
14	2.0	2.54	0.175	118.48		1.347
15	2.5	2.38	0.175	112.53		1.394
16	3.0	2.14	0.175	114.12	57.2°	1.328
17††	0.1	2.77	0.175	123.15		1.311
18	0.5	2.75	0.175	14.384		1.335
19	0.75	2.74	0.175	124.26		1.288
20	1.0	2.70	0.175	126.4		1.325
21	1.5	2.63	0.175	128.87		1.317
22	2.0	2.54	0.525	114.22		1.35
23	2.5	2.38	0.175	112.53		1.394
24	3.0	2.14	0.175	100.96		1.392
25	3.5	2.05	0.175	97.2		1.422
26	2.5	2.38	0.525	114.22	57.2°	1.25
27	2.5	2.38	0.0044	108.77	58°	1.274
28	2.5	2.38	1.75	122.11	57.2°	1.332
29	2.5	2.38	0.0175	108.77	58°	1.597

*The rest specimens were used as new cases for 'true' prediction.
†The data was used to test the model learned.
††The data was used to recognize the model.

Table II. The recognized values for parameters in strain–stress–time constitutive model.

	Best fitness	a	m	f	β	c
Pre-peak strength (σ_1–σ_3)	0.0177466	1.25	0.03906	0	5.32227	0.00488
Post-peak strength (σ_1–σ_3)	0.0462756	1.13769	0	0.92773	1.85546	1.88476

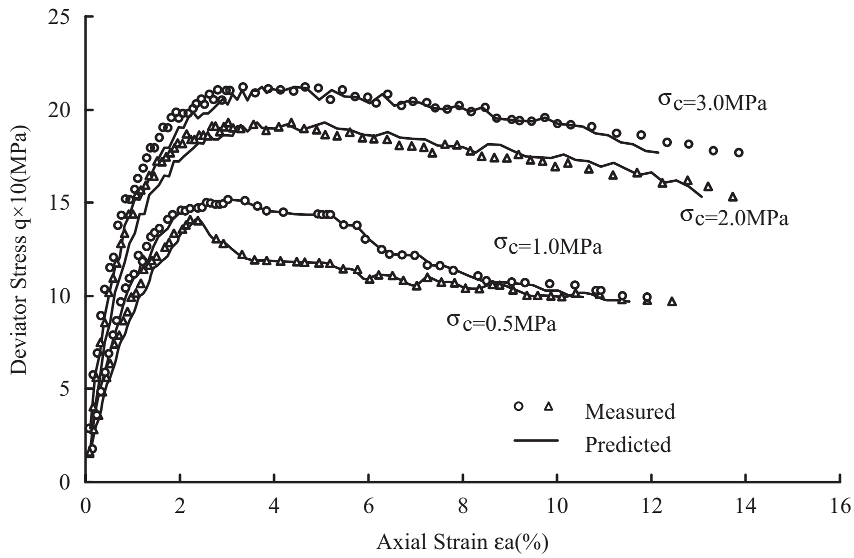


Figure 7. Comparison of generalization predictions of the recognized constitutive model and measurements for the intact specimen, $\dot{\varepsilon}_a = 0.175\%/\text{min}$ for $\sigma_c = 0.5$, $\sigma_c = 1.0$, and $\sigma_c = 3.0$, and $\dot{\varepsilon}_a = 0.525\%/\text{min}$ for $\sigma_c = 2.0$, the data was not used for building the constitutive model.

determined as number of generations $Z = 10$, jump crossover probability $P_c = 0.95$, creep mutation probability $P_m = 0.02$, inversion mutation probability $P_i = 0.2$, and gap $G = 100\%$.

These data measured at the consolidated and undrained triaxial tests are randomly divided into three groups. One is used as fitness case to obtain constitutive model, indicated by '††' in Table I. Another is used as new cases to test generalization capability of the learned model, indicated by '†' in Table I. The rest in Table I are used as new cases for 'true' predictions.

The parameter values for the coefficients a, m, f, β and c in Equations (5) and (7) were reasonably recognized (see Table II). The model gave its accurate learning for this specimen with error of 2.49% (see Figure 5).

6. PREDICTIONS OF THE LEARNED CONSTITUTIVE MODEL

Another important issue is the generalization capability of the solution: How good is the resulting model when used with experimental conditions that are similar with or

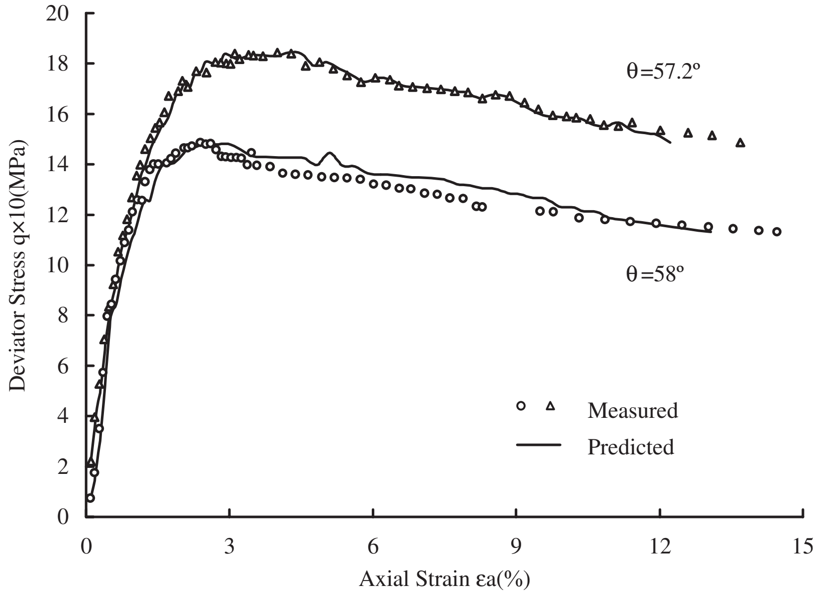


Figure 8. Comparison of generalization predictions of the recognized constitutive model and measurements for pre-cut specimens with $\sigma_c = 2.5$ MPa, $\dot{\varepsilon}_a = 0.552\%/\text{min}$ for $\theta = 57.2^\circ$ specimen and $\dot{\varepsilon}_a = 0.0044\%/\text{min}$ for $\theta = 58^\circ$ specimen, the data was not used for building the constitutive model.

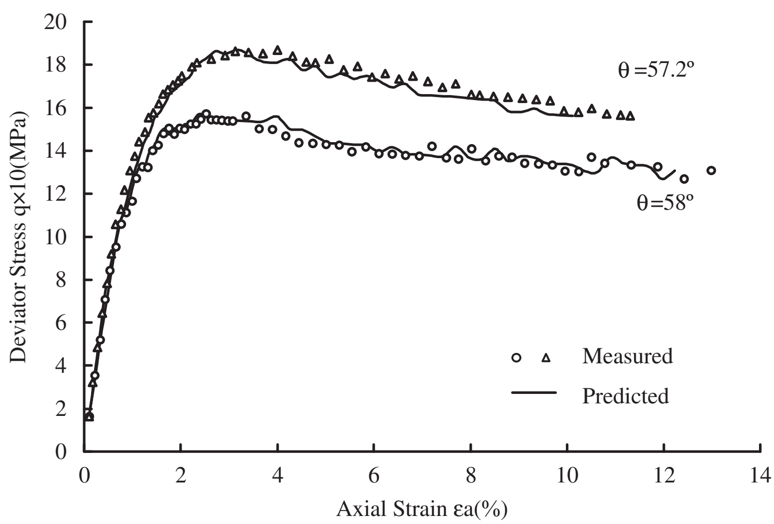


Figure 9. Comparison of generalization predictions of the recognized constitutive model and measurements for precut specimens with $\sigma_c = 2.5$ MPa, $\dot{\varepsilon}_a = 1.75\%/\text{min}$ for $\theta = 57.2^\circ$ specimen and $\dot{\varepsilon}_a = 0.0175\%/\text{min}$ for $\theta = 58^\circ$ specimen, the data was not used for building the constitutive model.

different from those used during the identification process? The answer to that question can in turn give some advantage to complex but robust representations over simpler but unstable ones.

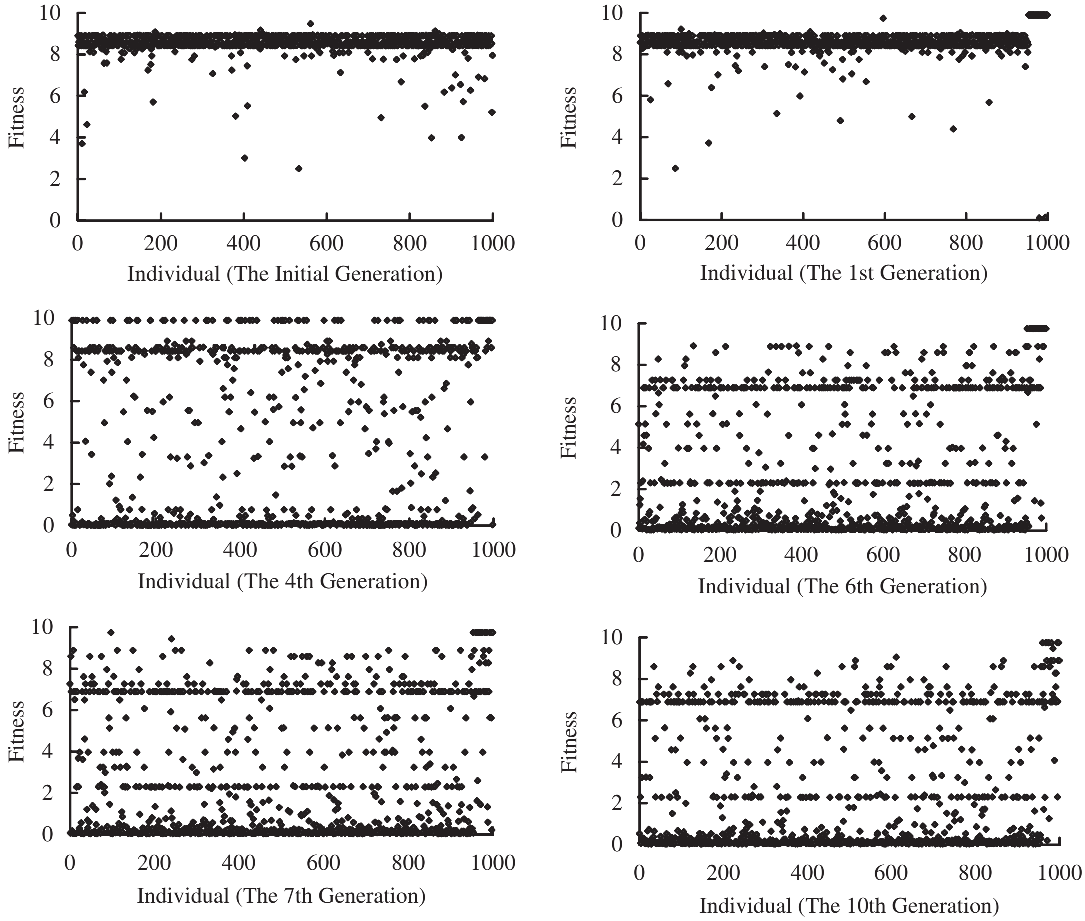


Figure 10. Change of individuals as evolutionary for pre-peak strength model.

The computation of the fitness should take the generalization issue into account: one usually considers during the identification process more than one experimental condition, here pre-peak and post-peak, also termed fitness cases. The fitness is then average of the error over all fitness cases. The recognized model was used to predict behaviours of other similar soils with different consolidated pressure values between 0.1 and 3.5 MPa. Some results are shown in Figures 6 and 7. The model gave excellent predictions for intact samples under varying consolidated pressures.

The model obtained from the intact soil data was also used to predict behaviours of soil with precut with different angles under varying consolidated states with pressure values 0.1–3.5 MPa. The predictions for non-linear behaviours of soil specimen with a precut of 57.2–58° under consolidated pressure of 2.5 MPa are only in agreement with their measurements (see Figures 8 and 9). It indicates that the constitutive model obtained from the soil sample without precuts under consolidated pressure of 0.1 MPa cannot be used in most cases of precut.

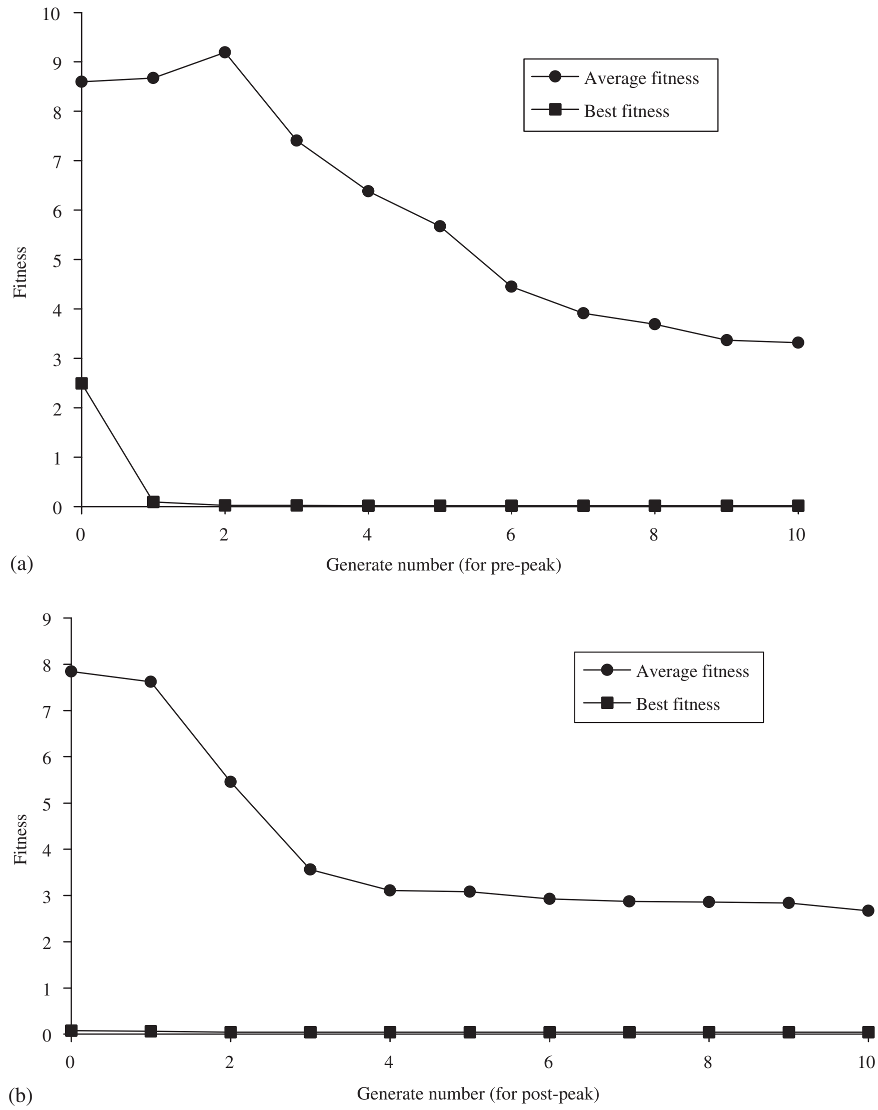


Figure 11. Change of fitness as generation number of evolution: (a) for pre-peak; and (b) for post-peak.

7. DISCUSSION AND CONCLUSIONS

For highly non-linear soils, recognition of the constitutive model from experimental data has a large parameter space and is highly multimodal. The approach shall be robust enough and efficient enough in global optimum. A genetic evolutionary approach, a global optimal approach, is thus proposed for recognition of non-linear constitutive models of a soil with a given structure. The coefficients in the non-linear model are learned from experimental data using genetic evolutionary approach. In the recognition press, the initial coefficient sets

Table III. The best individual at each evolutionary generation.

Generation no.	The best people no.	The minimum fitness	a	m	f	β	c
Pre-peak strength $(\sigma_1–\sigma_3)$							
0	532	2.49137	5.12207	1.95313	4.366523	1.06445	7.21191
1	979	0.0953485	0.01953	0	0	5.00977	0
2	732	0.022104	0.01953	0.01953	0	0.09765	5
3	18	0.0210963	0.01953	0.01953	0	0.09765	0
4	923	0.0179771	0.15625	0.03906	0	0.67383	0.3125
5	991	0.0179771	0.15625	0.03906	0	0.67383	0.3125
6	986	0.0179771	0.15625	0.03906	0	0.67383	0.3125
7	910	0.0177466	1.25	0.03906	0	5.32227	0.00488
8	70	0.0177466	1.25	0.03906	0	5.32227	0.00488
9	962	0.0177466	1.25	0.03906	0	5.32227	0.00488
10	191	0.0177466	1.25	0.03906	0	5.32227	0.00488
Post-peak strength $(\sigma_1–\sigma_3)$							
0	249	0.0786414	1.13769	0.01953	1.24023	2.40234	1.88964
1	521	0.0648654	2.38769	0.01953	1.04492	3.33007	6.88964
2	423	0.0462756	1.13769	0	0.92773	1.85546	1.88476
3	340	0.0462756	1.13769	0	0.92773	1.85546	1.88476
4	145	0.0462756	1.13769	0	0.92773	1.85546	1.88476
5	36	0.0462756	1.13769	0	0.92773	1.85546	1.88476
6	181	0.0462756	1.13769	0	0.92773	1.85546	1.88476
7	34	0.0462756	1.13769	0	0.92773	1.85546	1.88476
8	361	0.0462756	1.13769	0	0.92773	1.85546	1.88476
9	694	0.0462756	1.13769	0	0.92773	1.85546	1.88476
10	641	0.0462756	1.13769	0	0.92773	1.85546	1.88476

randomly created are used to predict non-linear behaviours of the learning and testing cases. The error between prediction of each tentative model and measurement is calculated to evaluate its generalization capability. Then the models are evolved into next generation to create a set of new tentative models. The created tentative models are undergone the same evaluation as previous generation. The error decreases as progress of the evolutionary process. The process continues until the best model having minimum error is found. For example, during recognition of pre-peak model, the fitness of individuals at 0th and 1st generations is almost about 0.8–0.9. As evolutionary, more and more individuals concentrated to the minimum fitness close to zero (Figure 10). The minimum fitness of 0.0177466 was found at 7th generation for pre-peak model and 0.0462756 was found at 2nd generation for post-peak model (Figure 11 and Table III). Therefore, 10 generations are enough to find reasonable solution for soil recognition.

Through this evolution process, the constitutive model for describing the time dependency of diatomaceous soil under consolidated and undrained state both for pre- and post-peak strength process was found as

$$\mathrm{d}\varepsilon = \frac{a}{4.605\beta}\left[1 + \sqrt{1 - \frac{9.21}{a}(C'_{21}\,\mathrm{d}p' + C'_{22}\,\mathrm{d}q)}\right]$$

$$C'_{21} = m\frac{\chi'}{Mp' \times 10^{-1}}, \qquad C'_{22} = f\frac{\chi'}{M^2 p' \times 10^{-1}[1 + \ln(p'/p'_0)]}, \quad \chi' = \frac{\lambda - k}{1 + ce_0}$$

For the different state, pre- and post-peak strength process, the only difference is values of the coefficients in the equation. The recognized results for pre-peak strength process are $a = 1.25$, $m = 0.03906$, $f = 0.0$, $\beta = 5.3227$, and $c = 0.00488$, respectively. However, $a = 1.13769$, $m = 0.0$, $f = 0.92773$, $\beta = 1.85546$, and $c = 1.88476$ are recognized for post-peak strength process.

Since the constitutive model is learned from experimental data. The experimental data both for learning and testing cases should be representative at indicating essence of non-linear behaviours of materials. It needs not only learning cases but also testing cases. Use of the testing cases can avoid 'lack-learning' or 'over-learning' problem. Therefore, it needs at least two sets of the entire stress vs strain curve obtained from mechanical test. One is used for fitness cases and another is used for testing case. The constitutive model in the paper is recognized only using stress–strain data of an consolidated soil with consolidated pressure of 0.1 MPa and gave good predictions for all other consolidated pressures (Figures 6–7). It indicates that the recognized non-linear constitutive model captured the intrinsic characteristics and has good generalization capability for predicting non-linear time-dependency behaviour of diatomaceous soil at different consolidation pressures. The proposed method can be used to construct automatically soil constitutive models once the underlying response mechanism has been determined.

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