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Sparse Polynomial Chaotic Expansion for Uncertainty Analysis of Tunnel Stability

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ABSTRACT

Uncertainty is an intrinsic property of rock engineering because of the complicated geology conditions, rock failure mechanism ambiguity, and the nonlinear mechanical behavior of surrounding rock mass. We developed a novel framework to handle the uncertainty by combing the Sparse polynomial chaotic expansion (SPCE), numerical model, and reliability method. The SPCE model was used to map the complex relationship between the response of the surrounding rock mass and its uncertainty. The first-order reliability method (FORM) evaluated the reliability index and failure probability. Based on the SPCE model and FORM, a simple global optimization algorithm (SHGO) seeks design points and corresponding reliability indexes. A circular tunnel verified the developed framework with a close-form solution. The reliability index, design point, and failure probability were in excellent agreement with the FORM and Monte Carlo simulation. This indicated that the SPCE model could be used as a surrogate model for the analytical solution to approximate the tunnel response (including deformation and size of the plastic zone). Then, the developed framework was employed in a horseshoe tunnel by combing with the numerical model. The results further proved that the developed framework is feasible and effective for handling uncertainty in rock engineering. Furthermore, the developed framework is effective, efficient, and accurate for reliability analysis and provides a helpful tool to approximate the response of rock structure to avoid the time-consuming numerical model in practical rock engineering.

1. Introduction

Uncertainty is an intrinsic attribute of rock mechanics and rock engineering and has become one of the critical factors affecting rock mass instability and failure. In the past decades, uncertainty has attracted increasing attention in rock mechanics and rock engineering (Panthi and Nilsen, 2007; Fellin et al., 2010; Tiwari et al., 2017; Zhao et al., 2021b, 2021c; Zhang et al., 2022). However, uncertainty analysis has not been adequately applied to rock engineering, including geological survey, scheme planning, stability analysis, design, and construction. The reliability approach is a helpful tool that has broadly been utilized to solve the uncertainty problem in engineering systems (Hoek, 1998; Griffiths et al., 2011; Zhao et al., 2014).

In the past decades, various reliability approaches, such as the first-order reliability method (FORM), the second-order reliability method (SORM), and Monte Carlo simulation (MCS) (Ditlevsen and Madsen, 1996; Zhao and Ono, 1999; Lv and Low, 2011), have been proposed and applied to rock engineering. Limit state function is an integral part of the reliability approach, and its derivative information is the key to determining the reliability index in reliability approaches. Numerical methods have been widely used in rock mechanics to calculate surrounding rock response (including deformation and stress) (Jing and Hudson, 2002). However, it is challenging to obtain explicit limit state function and its derivative information in practical rock engineering using the numerical method, which hinders the application of the reliability approach in practice. MCS (Ditlevsen and Madsen, 1996), importance sampling (IS) (Hsu and Ching, 2010), subset simulation (SS) (Au and Beck, 1999), and other simulation

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methods have also been developed to overcome the above-mentioned problem. However, the simulation-based methods are costly, which limits their application in practical and large-scale engineering. Various surrogate models and the response surface method (RSM) were established instead of the numerical model to improve the efficiency of reliability analysis (Li et al., 2016; Zhao et al., 2021a).

RSM provides an approximate limit state function for reliability analysis in rock engineering. The polynomial RSM was employed to analyze the reliability of underground rock excavation using SORM (Lv and Low, 2011). The Kriging-based RSM was established and successfully applied to evaluate the stability of geotechnical engineering problems under uncertainty (Luo et al., 2012). The multiplicative dimensional reduction method was utilized to approximate tunnel response and consider the uncertainty (Zhao et al., 2021c). The high dimensional model representation was also applied to approximate the limit state function and to evaluate the uncertainty of geotechnical engineering problems under the uncertainty (Chowdhury and Rao, 2010). With the development of machine learning, various machine learning methods, including neural networks, support vector machines, and random forests, were used to generate the RSM for the reliability analysis of geotechnical and geological engineering problems (Deng et al., 2005; Zhao, 2008; Cho, 2009; Kang and Li, 2016; Zhao, 2017). Tan et al. (2011) adopted support vector machines and neural networks for evaluating the reliability of geotechnical engineering respectively and compared their performance. In practice, the surrogate model is an integral part of the reliability evaluation of geotechnical engineering. The development of the surrogate model enhanced the efficiency of the reliability analysis, especially for large-scale practical rock engineering problems. However, the traditional surrogate model does not directly capture the uncertainty of the engineering systems.

The idea of the polynomial chaotic extension model (PCE) was introduced under the background of uncertainty quantization and was often considered as an excellent surrogate model due to its global convergence (Xiu and Karniadakis, 2002). PCE provides an excellent tool to map the relationship between uncertainty and the corresponding response of the engineering systems. However, with the increase of uncertain variables and PCE order, the PCE coefficient will increase significantly. The sparse polynomial chaotic expansion (SPCE) was established to avoid the above-mentioned disadvantage of the PCE (Blatman and Sudret, 2010). The optimal technology is another integral part of the reliability analysis for FORM. We used simplicial homology global optimization (SHGO) to determine the design points and the corresponding reliability indexes based on FORM. Finally, a novel reliability analysis framework was established by combining the SPCE models, SHGO, FORM, and numerical models.

This study employed the experimental design to construct the input of samples and obtain the samples based on a numerical model. The SPCE was established based on samples and replaced the numerical model as a surrogate model in reliability analysis. Then, reliability analysis was implemented based on the SPCE model and FORM. The organization of this study is as follows: First, the sparse polynomial chaotic expansion is introduced in Section 2. Second, the idea and procedure of the SPCE-based surrogate model are briefly represented in Section 3. Then the developed framework is validated by using a circular tunnel with a closed-form solution in Section 4. In Section 5, the stability of a horseshoe tunnel was studied under uncertainty based on a numerical model using the developed framework. Finally, Section 6 summarizes this study and draws some conclusions.

2. Sparse Polynomial Chaotic Expansion

The original PCE method was introduced as a surrogate model to replace the complex and time-consuming deterministic model in practical engineering. In this study, SPCE, an extension of the original PCE, was used to replace the numerical model under a non-intrusive framework, significantly improving the calculation efficiency. An analytical equation of joint probability density functions is gained based on orthogonal multidimensional polynomials of input vectors using SPCE. This study took advantage of the Gaussian joint probability density function by combining it with Hermite multivariate polynomials. In practical applications, a recursive relationship is utilized to construct the one-dimensional Hermitian polynomials as follows:

$$H_0(x) = 1\,, \tag{1}$$

$$H_{n+1}(x) = xH_n(x) - nH_{n-1}(x)\,, \tag{2}$$

where $H_n(x)$ denotes the Hermite function, and x represents the variables. The Gaussian probability density function is orthogonal to the Hermite polynomials function.

$$\int_{-\infty}^{+\infty} H_m(x)H_n(x)\varphi(x)dx = n!\delta_{mn} \tag{3}$$

where $\varphi(x)$ stands for the probability density function of a random variable with standard normal distribution.

Generally, the response (deformation, plastic zone, etc.) induced by excavation Γ depends on the M random variables (mechanical parameters, in-situ stress, and boundary conditions) in the tunnel. A p order PCE can represent the response Γ based on SPCE as follows:

$$\Gamma_{PCE}(\xi) = \sum_{\beta=0}^{\infty} \alpha_\beta \Psi_\beta(\xi) \cong \sum_{\beta=0}^{P-1} \alpha_\beta \Psi_\beta(\xi)\,, \tag{4}$$

where ξ signifies an M dimensional independent random variables vector with standard normal distribution; P symbolizes the number of the truncation terms and $P = \frac{(M+p)!}{M!\,p!}$; α_β indicates the unresolved PCE coefficient, which can be determined using the regression technology later. ψ_β denotes multidimensional Hermite polynomial. The product of one-dimensional Hermite polynomials of different random variables (Eqs. (1) and (2)) is utilized to obtain multidimensional Hermite polynomials as follows:

$$\Psi_{\beta} = \prod_{i=1}^{M} H_{\alpha_i}(\xi), \tag{5}$$

where α_i stands for an M non-negative integers sequence $\{\alpha_1, \alpha_2, \ldots, \alpha_M\}$ and $H_{\alpha i}(.)$ represents the α_i th one-dimensional Hermite polynomial. The Hermite polynomial can be represented as follows:

$$\Gamma_{\alpha_i}(x_{i_1}, x_{i_2}, \cdots, x_{i_M}) = (-1)^{\alpha_i} e^{1/2X^T X} \frac{\partial^M}{\partial x_{i_1} \partial x_{i_2} \cdots \partial x_{i_M}} e^{-1/2X^T X}, \tag{6}$$

where X (i.e., $(x_{i1}, x_{i2}, ..., x_{iM})$) signifies the standard normal random variables.

For the truncation scheme of PCE in Eq. (4), the number of unsolved coefficients increases substantially with the increase of the random variables number and the PCE order, and it will inevitably hamper the application of PCE in practical engineering. The SPCE was employed to reduce the number of PCE truncated terms based on the low-rank and hyperbolic truncation scheme (Blatman and Sudret, 2011). Sparse PCE models that exclude non-significant terms have excellent performance (Blatman and Sudret, 2010). The so-called q quasi-norm can be used to represent SPCE as follows:

$$\|\alpha\|_q = (\sum_{i=1}^{M} (\alpha_i)^q)^{1/q} \le p, \tag{7}$$

where q indicates a coefficient, which can be arbitrarily selected in the interval $(0 < q < 1)$ based on the above equation. Blatman and Sudret (2011) proved that the accuracy is enough under $q \ge 0.5$. SPCE induces the number of unresolved coefficients dramatically. An iterative algorithm was proposed to construct the SPCE model (Blatman and Sudret, 2010).

3. SPCE-Based Uncertainty Analyses

In engineering systems, the reliability approach is an effective tool to handle uncertainty. The reliability approach, FORM, was utilized in this study to compute the reliability index of the tunnel stability. Optimization technology and limit state function are two integral components in the FORM procedure. SHGO was selected as the optimal tool to find the reliability index based on FORM. SPCE model was utilized to approximate the limit state function based on the numerical model, enhancing the performance of reliability analysis. Fig. 1 shows the framework of reliability analysis based on SPCE.

3.1 Reliability Analysis Method

FORM has been commonly employed in various engineering fields for handling uncertainty. A practical and efficient FORM was developed for geotechnical engineering problems by combing the Hasofer-Lind index and MS Excel Solver (Low, 2004). The reliability index was procured by solving the constrained optimization problem based on the improved FORM and the extended random variables ellipsoid in the original space. Meanwhile, the dimensionless number n_i was introduced to avoid calculating normal distribution equivalent means and standard deviations (Low and Tang, 2007).

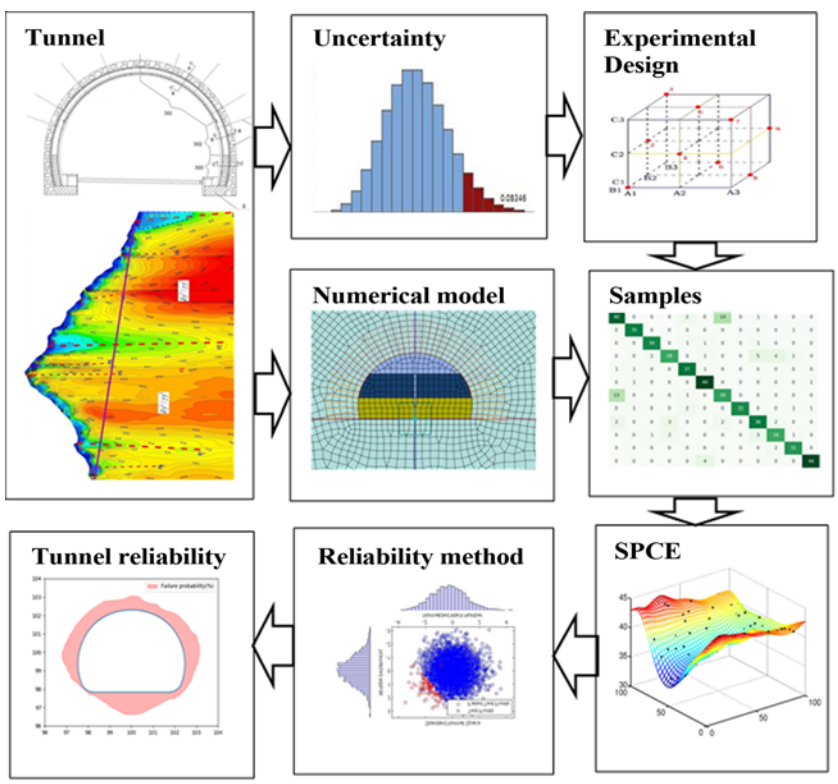

Fig. 1. The Framework of the SPCE-Based Reliability Analysis

$$\beta = \min_{X \in F} \sqrt{[\mathbf{n}]^T [R]^{-1} [\mathbf{n}]} \tag{8}$$

where **n** and R signify a column vector of n_i and the correlation matrix, respectively. In the unconstrained optimization (Eq. (8)), the relationship between n_i and the corresponding random variables x_i can be represented in the following form:

$$x_i = F^{-1}[\phi(n_i)] . \tag{9}$$

We selected SHGO as an optimal method to find the reliability index according to the FORM (Eq. (8)). The solving procedure of the reliability index was implemented and coded in Python.

3.2 SHGO

SHGO is a universal global optimization technology based on combinatorial topology and simple integral homology. The SHGO algorithms, which only compute the value of the objective function and do not need derivative information, are suitable for black-box optimization problems. The algorithm includes four phases generating uniform sampling vertices, constructing the directed simplicial complex, constructing the minimizer pool based on Sperner's lemma (Sperner, 1928), and conducting local minimization based on the starting points. We adopted SHGO to handle the optimization problem in FORM. The idea, algorithm, and procedure were briefly described by Endres et al. (2018).

3.3 SPCE-Based Surrogate Model

The uncertainty analysis is costly and time-consuming, and even it is impossible to implement it in an actual tunnel because it is not easy to obtain the analytical solution for tunnel excavation. The numerical solution is broadly used in design, stability analysis, and construction in practical engineering. We used the SPCE model instead of the numerical model to generate the limit state function. The limit state function was generated based on the SPCE-based surrogate model. In the uncertainty analysis of the tunnel, the SPCE model (Eq. (4)) was employed to obtain the response of the surrounding rock mass. The SPCE model presented the complex relationship between the response of the surrounding rock mass and the corresponding random variable in the tunnel as follows:

$$SPCE(\boldsymbol{X}):\ R^N \to R\ , \tag{10}$$

$$y = \mathrm{SPCE}\ (\boldsymbol{X}), \tag{11}$$

where $\boldsymbol{X}$ denotes a vector of random variables (such as Elastic modulus, internal friction angle, or in-situ stress), i.e., $\boldsymbol{X} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N)$, $\boldsymbol{x}_i(i = 1, 2, \ldots, N)$; and $\boldsymbol{y}$ indicates the response of the surrounding rock mass in the tunnel (such as deformation, plastic zone, and stress). To obtain SPCE($\boldsymbol{X}$), the SPCE model was generated based on the procedure described in Section 2 using Python 3.0.

3.4 Determining the Limit State Function

Determination of limit state function is an integral component in the uncertainty analysis. In order to evaluate the uncertainty of tunnel stability, the displacement and plastic zone were chosen as the index of tunnel stability in this study. The limit state function for the tunnel uncertainty analysis is in the following form:

$$Z = d(X_1, X_1, \ldots, X_n) - d_{lim}\ , \tag{12}$$

where $d(X_1, X_1, \ldots, X_n)$ is the inward displacement of the tunnel wall, which is determined by SPCE and dis_{lim} is the permitted limit value of inward displacement for the tunnel wall.

3.5 Determining the Reliability Index or Failure Probability

In this study, FORM (Eqs. (8) and (9)) was used to compute the reliability index and evaluate failure probability. SHGO was employed to deal with the optimal problem (Eq. (8)) based on the SPCE model in the Python Scipy optimization package.

3.6 The Procedure of the Developed Method

The SPCE model was utilized as a surrogate model to generate the limit state function instead of the numerical model in the reliability analysis. The excavation response of the tunnel, such as deformation and stress, was predicted by the above-described SPCE model. SHGO was chosen as the optimal technology for FORM. Then the failure probability and reliability index were computed using the developed framework, which combined the reliability approach, SHGO, and the SPCE model. The procedure of implementation (Fig. 2) is as follows:

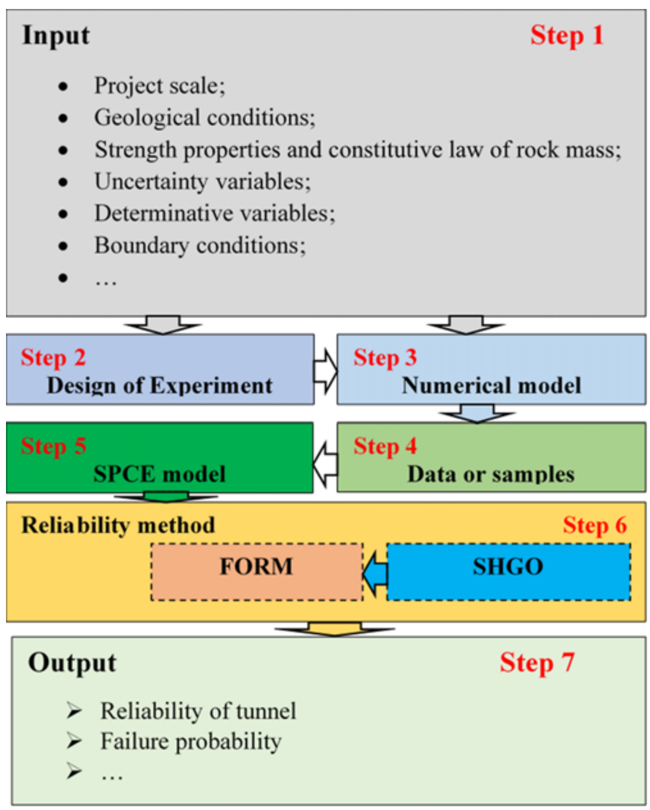

Fig. 2. The Flowchart of the Developed Method

Step 1: Collect geological engineering conditions, engineering scale, strength characteristics, constitutive law of surrounding rock mass, uncertain variables, determining variables, boundary conditions, and other engineering data.

Step 2: According to the data obtained in the above steps, tentative combinations of uncertain variables are determined through experimental design.

Step 3: Establish a numerical model according to the engineering scale, strength characteristics, rock mass constitutive law, and boundary conditions obtained in Step 1.

Step 4: According to the numerical model established in the previous step, the tunnel response of the tentative combination of uncertain variables is calculated.

Step 5: According to the algorithm described in Section 2, determine the PCE coefficient and obtain the SPCE model.

Step 6: Call the FORM to conduct the reliability analysis and obtain the reliability index and the failure probability using the SPCE model and the SHGO.

Step 7: Evaluate the tunnel under uncertain conditions for stability analysis, design, enforcement, construction, etc.

4. Validation

We utilized a circular tunnel under hydrostatic stress to demonstrate and verify the established framework. We assumed that the surrounding rock mass is continuous, isotropic, and homogeneous. The initial rock mass was assumed to be under the action of support pressure p_i and hydrostatic far-field stress p_0 (Fig. 3). When the critical pressure p_{cr} exceeds the support pressure p_i, a plastic zone will appear because of stress adjustment induced by excavation. The corresponding inward displacement and size of the plastic zone of the tunnel wall were approximated using the SPCE surrogate model. According to rock mechanics theory, the inward displacement of tunnel wall u_{ip} and the plastic zone size r_p can be calculated as follows:

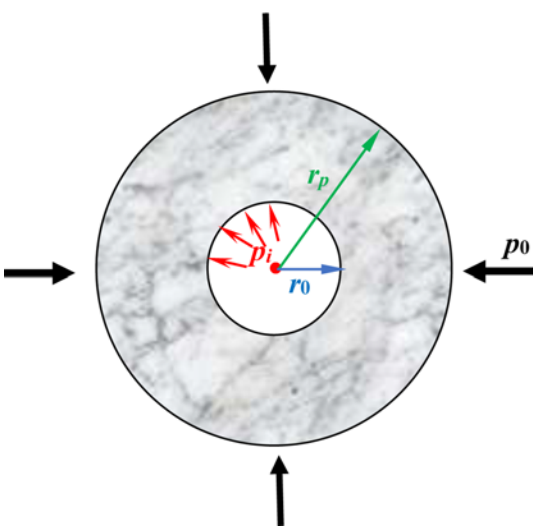

Fig. 3. A Circular Tunnel Under Hydrostatic Stress and Support Pressure

Table 1. Statistical Parameters of Random Variables (Zhao et al., 2014)

Rock properties	Mean value	Standard deviation
E/Mpa	373	48
c/Mpa	0.23	0.068
φ/°	22.85	1.31

$$\frac{r_p}{r_0} = \left[\frac{2(p_0+s)}{(k+1)(p_i+s)}\right]^{\frac{1}{k-1}} \tag{13}$$

$$\frac{u_{ip}}{r_0} = \left[\frac{1+\upsilon}{E}\right]\left[2(1-\upsilon)(p_0-p_{cr})\left(\frac{r_p}{r_0}\right)^2 - 2(1-2\upsilon)(p_0-p_i)\right] \tag{14}$$

where E and υ indicate the elastic modulus and Poisson's ratio, respectively. The value of p_{cr}, k, s, and σ_c can be determined based on the strength parameters of the rock mass. The detailed mechanical model and its derivation (Eqs. (13) and (14)) can be found in Duncan Fama's study (Duncan Fama, 1993).

Elastic modulus E and strength parameters (including cohesion c and internal friction angle φ) were regarded in this study as the uncertainty variables with a normal distribution. The statistical parameters are shown in Table 1. Furthermore, we assumed that φ is negatively correlated with c and the correlation coefficient is -0.5. Other parameters were the deterministic values.

The uncertainty of surrounding rock displacement and the plastic zone was evaluated to demonstrate and verify the established framework. The radius of the plastic zone can be acquired based on Eq. (10). Table 1 lists the statistical index of random variables. The radius of the plastic zone should be uncertain because of the uncertainty of the surrounding rock mass strength and mechanical parameters. The inputs of uncertain variables (i.e., elastic modulus, cohesion, and internal friction angle) were assumed to have a normal distribution. To establish the SPCE model, a Latin hypercube sampling method was utilized to generate tentative combinations of 50 uncertain variables. The responses of each tentative combination were calculated using the analytical solutions (Eqs. (10) and (11)). Therefore, 50 samples were generated based on a tentative combination of 50 uncertain variables and corresponding tunnel responses. Based on 50 samples, the SPCE model was constructed by using the corresponding algorithm in Section 2. The comparison of the tunnel response (deformation and plastic zone size) predicted by SPCE and calculated by the analytical solution is shown in Fig. 4. The predicted tunnel response by SPCE is almost identical to that of the closed-form solution. The results showed that the SPCE model could capture the mechanical behavior and failure mechanism of the rock mass caused by the excavation. The relationship between the distance from the tunnel center and deformation is exhibited in Fig. 5. The results show that the surrounding rock deformation predicted by SPCE agreed well with the theoretical deformation law. It was further proved that the SPCE model can capture the mechanical behavior of tunnel surrounding rock.

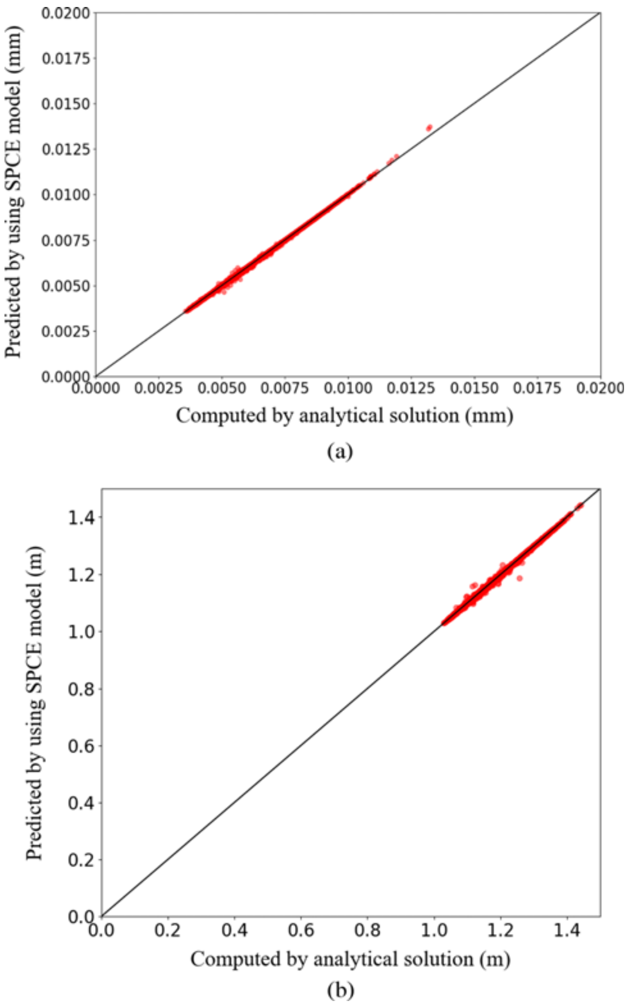

Fig. 4. The Comparison between the Analytical Solution and SPCE: (a) Displacement, (b) Plastic Zone

Once the SPCE model was finished, the reliability of the tunnel was evaluated based on FORM. The limit state functions are essential to evaluate tunnel stability under uncertain conditions using the reliability approach. We adopted the following limit state functions.

$$g_1(x) = L - \frac{r_p}{r_o}, \tag{15}$$

$$g_2(x) = \varepsilon_L - \frac{u_{ip}}{r_o}, \tag{16}$$

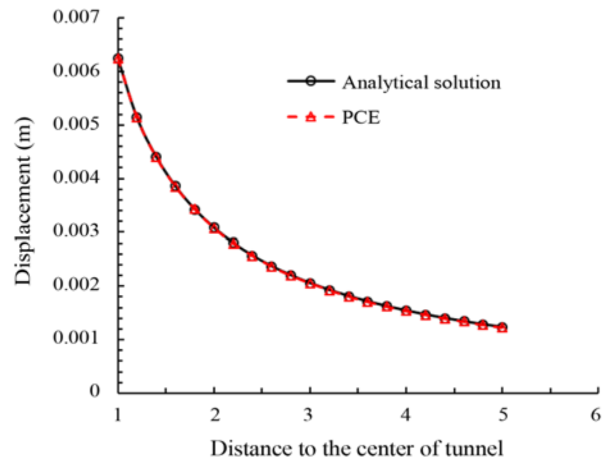

Fig. 5. The Relationship between Deformation and the Distance to the Center of the Tunnel

where L and ε_L signify the allowable thresholds of the plastic zone and tunnel wall inwards displacement, respectively. In this study, the L was considered to be equal to 3 and the ε_L equal to 0.01. p_0, p_i, r_0, and μ are the determinative variables, while Elastic modulus E, cohesion c, and internal friction angle φ are the uncertainty variables.

To demonstrate and validate the performance of the developed uncertainty analysis based on the SPCE, different limit state functions (Eqs. (15) and (16)) were adopted in tunnel uncertainty analysis. On this basis, the reliability index, design point, and failure probability were obtained (Tables 2 and 3). Their comparison is displayed in Figs. 6 and 7. The reliability indexes of the deformation zone and plastic zone based on the developed method were 3.3446 and 2.5632, respectively, and roughly the same as 3.3315 and 2.5977 obtained by the analytical solution-based FORM. The relative errors were smaller than 0.4% and 1.3%, respectively. The results demonstrated that the developed method was feasible for evaluating tunnel stability under uncertain conditions. The failure probability of the deformation and plastic zone based on the SPCE model was 0.047 and 0.52, respectively. They were almost identical to the analytical solutions (0.05 and 0.519). The relative errors were smaller than 6% and 0.12%, respectively. Meanwhile, the design points were almost identical to the analytical solution (Table 3 and Fig. 7). These results further proved that the developed method could rationally and effectively consider tunnel uncertainty, and the SPCE surrogate model could be utilized instead of the analytical solution.

Table 2. The Obtained Results in the Different Methods

		Deformation		Plastic zone	
		Reliability index	Failure probability	Reliability index	Failure probability
FORM	PCE	3.3446	0.0412	2.5632	0.519
	Analytic	3.3315	0.0432	2.5977	0.469
MCS	PCE	-	0.0470	-	0.520
	Analytic	-	0.0500	-	0.519

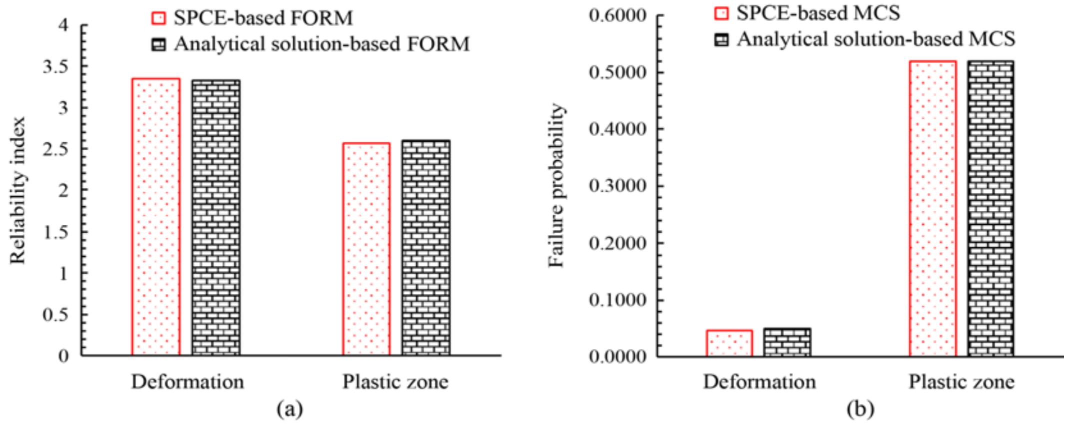

Fig. 6. The Uncertainty Comparison of the Different Methods: (a) Reliability Index, (b) Failure Probability

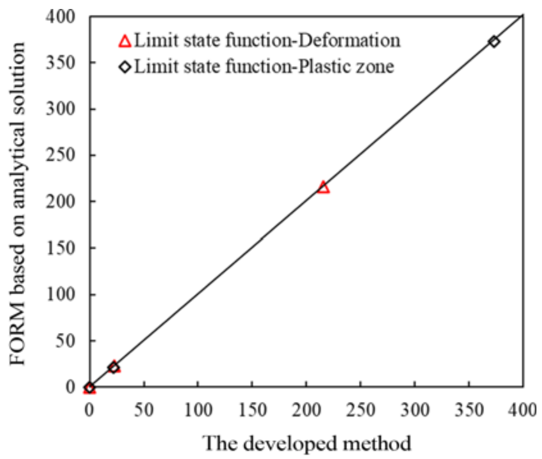

Fig. 7. The Design Point Comparison Based on the Different Methods

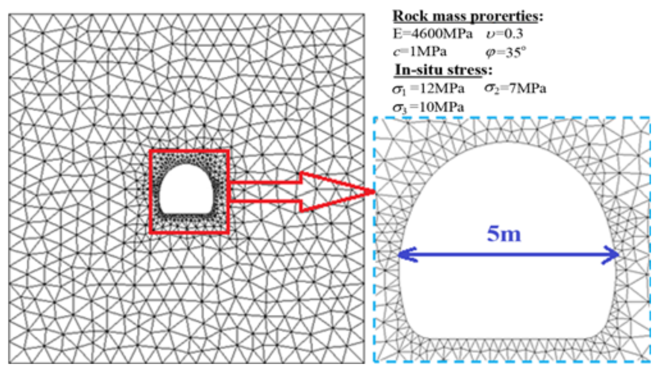

Fig. 8. The Geometry and Numerical Model of the Horseshoe Tunnel

5. Application

The developed uncertainty analysis method based on SPCE was applied to a horseshoe tunnel. Fig. 8 shows the geometrical size of the tunnel, the mechanical and strength parameters of rock, and in situ stress. The mechanical and strength parameters (including elastic modulus (E/MPa), internal friction angle (φ/°), cohesion (c/MPa)) and in situ stress (including major principal stress (σ_1/MPa), and minor principal stress (σ_2/MPa)) were considered as the uncertainty variables. Table 4 lists the statistical features of the uncertainty variables. The tunnel stability was evaluated by establishing a limit state function based on tunnel wall deformation under uncertainty. The tunnel finite element model was generated for the uncertainty analysis (Figs. 8 and 9). The numerical mesh is displayed in Fig. 8, made using a three-node triangular element. The number of elements was 1485, and the number of nodes was 822. The failure characteristics of the surrounding rock mass were simulated and modeled using the Mohr-Coulomb strength criterion. The fixed boundary conditions were utilized in this study. One hundred samples were generated based on the finite element model and Latin hypercube sampling. The SPCE model was established using the above samples. Fig. 10(a) shows the relative error of the 100 samples. We can observe that except for sample 13, the relative error for most of the samples was smaller than 10%. Fig. 10(b) exhibits the relative error after removing samples 13. The maximum relative error was

Table 4. Statistical Properties of the Uncertainty Variables in the Horseshoe Tunnel (Zhao et al., 2014)

	E(MPa)	c(MPa)	φ(°)	σ_1(MPa)	σ_3(MPa)
Mean value	4,600	1	35	10	7
Standard deviation	800	0.2	5	2	1.4

Table 3. Comparison of the Design Point

		Deformation			Plastic zone		
		E	c	φ	E	c	φ
FORM	PCE	215.2240	0.1991	22.6726	373.1246	0.1101	21.8949
	Analytical	215.7354	0.2003	22.6645	373.0000	0.1306	21.3720

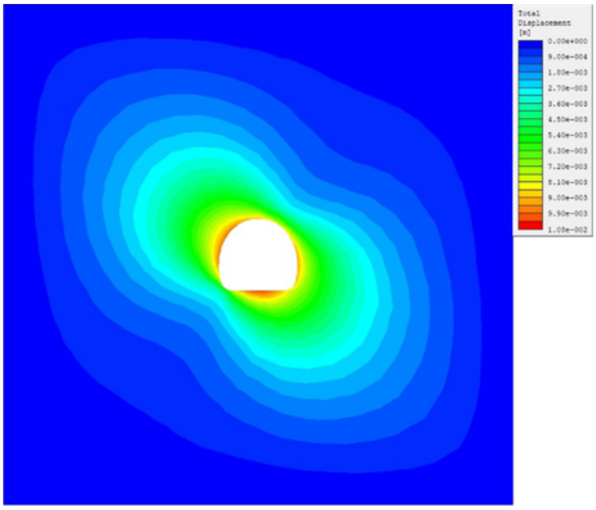

Fig. 9. The Numerical Model and Deformation Contours of the Tunnel at the Mean Value

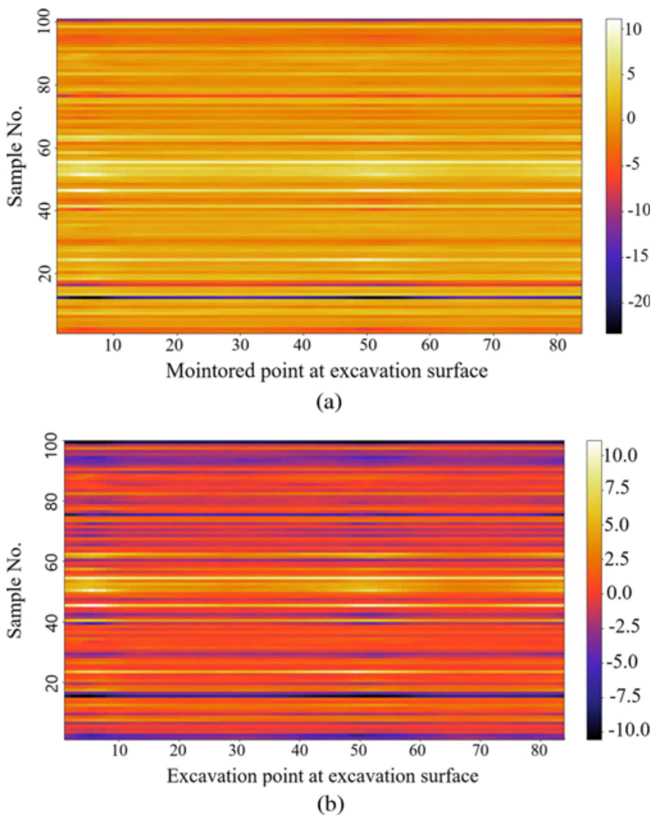

Fig. 10. The Relative Error of the Samples using SPCE: (a) All 100 Samples, (b) Exclusion of Sample 13

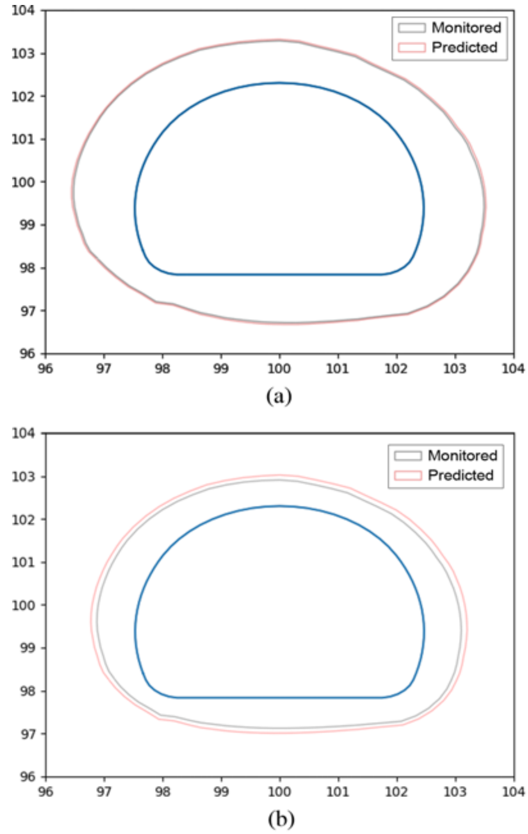

Fig. 11. The Comparison of Deformations Predicted by Numerical Simulation and PCE: (a) The First Sample, (b) Sample 13

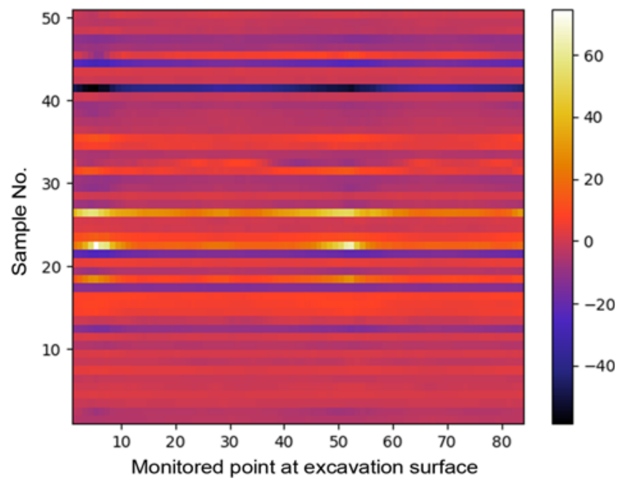

Fig. 12. The Relative Error of the Testing Sample using PCE

smaller than 10%. Using the numerical and SPCE model, the surrounding rock deformation induced by excavation is displayed as samples 1 and 13 in Fig. 11. The deformation obtained by the SPCE model was in excellent agreement with the numerical model, which conformed to the basic theory of rock mechanics. The results showed that the SPCE model could well capture the deformation characteristics of surrounding rock during excavation.

To further verify the performance of the SPCE model, we randomly generated 50 test samples different from the above 100 samples. The relative error of the predicted deformation is depicted in Fig. 12. Fig. 13 reveals the surrounding rock deformation of samples 1, 22, 26, and 41 and their comparison with the numerical model. We can observe that the deformations predicted by the SPCE model agreed with those of the numerical model. The maximum relative error was smaller than 20%. Therefore, the SPCE can meet the engineering requirements in the field of

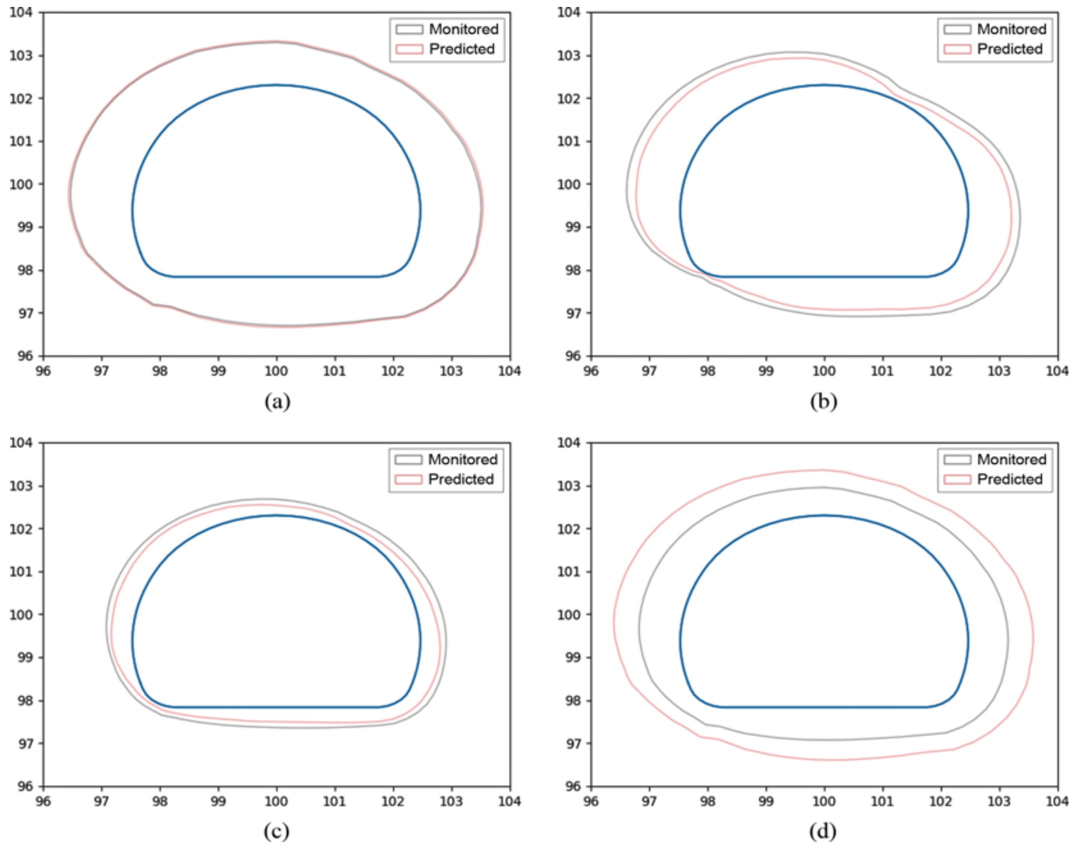

Fig. 13. The Comparison of Deformations Predicted by Numerical Simulation and PCE: (a) The 1st Sample, (b) The 22nd Sample, (c) The 26th Sample, (d) The 41st Sample

rock mechanics and rock engineering. Thus, the SPCE-based surrogate model can capture the deformation and mechanical behavior of tunnel surrounding rock instead of the numerical model.

The uncertainty of the tunnel was evaluated by the developed method based on the SPCE surrogate model. Table 5 shows the results of the uncertainty analysis of the horseshoe tunnels according to the different methods. The reliability index by the developed method was 2.065, which was very close to the 1.997 obtained based on the traditional response surface. The relative error was smaller than 3.5%. The design point was in good agreement with that of the traditional response surface method. The failure probabilities based on the traditional response surface, the developed method, and Monte Carlo simulation were 2.29%, 1.946%, and 3.28%, respectively. MCS was different from the developed methods due to SPCE generalization performance. In Table 5, there are 44 and 100 samples for the traditional polynomial response surface based on FORM (RSM-FORM) and PCE-based FORM. However, with the uncertainty variables number and polynomial order, the samples number will increase dramatically for the RSM-FORM. In the practical tunnel, there are more uncertainty variables and more complex limit state functions. Thus, the developed method can enhance the performance of the uncertainty analysis with better accuracy. Fig. 14 displays the

Table 5. The Uncertainty Analysis and Comparison Using the Different Methods

		RSM-FORM	PCE-FORM	PCE-MCS	Relative error (%)
Reliability index		1.997	2.065	-	3.405
Failure probability (%)		2.290	1.946	3.280	15.022
Design point	E(MPa)	3708.527	3627.993	-	2.172
	c(MPa)	1.061	0.978	-	7.814
	φ(°)	28.776	31.446	-	9.276
	σ_1(MPa)	12.043	12.819	-	6.442
	σ_3(MPa)	7.179	7.084	-	1.322
Number of computations		44	100	10,000	-

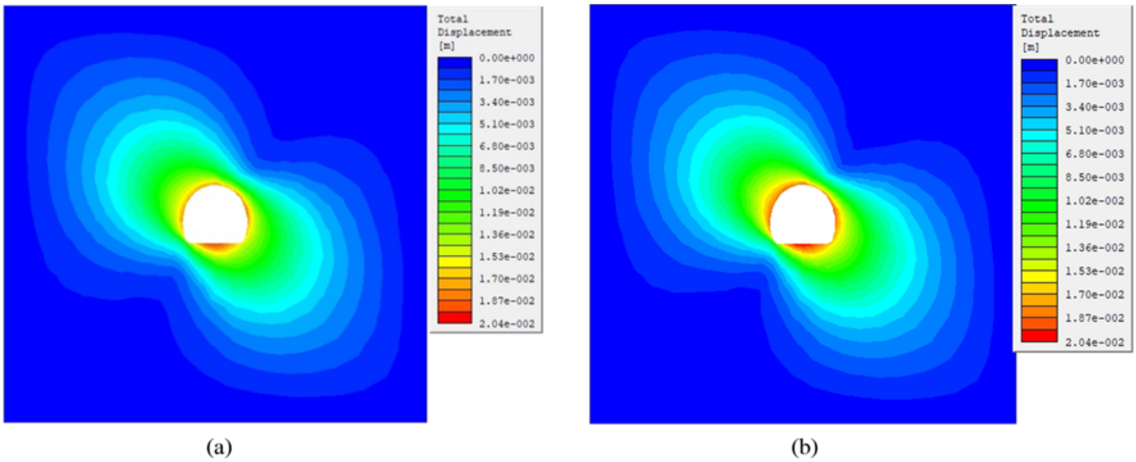

Fig. 14. The Deformation Contours Based on the Design Point: (a) RSM, (b) SPCE

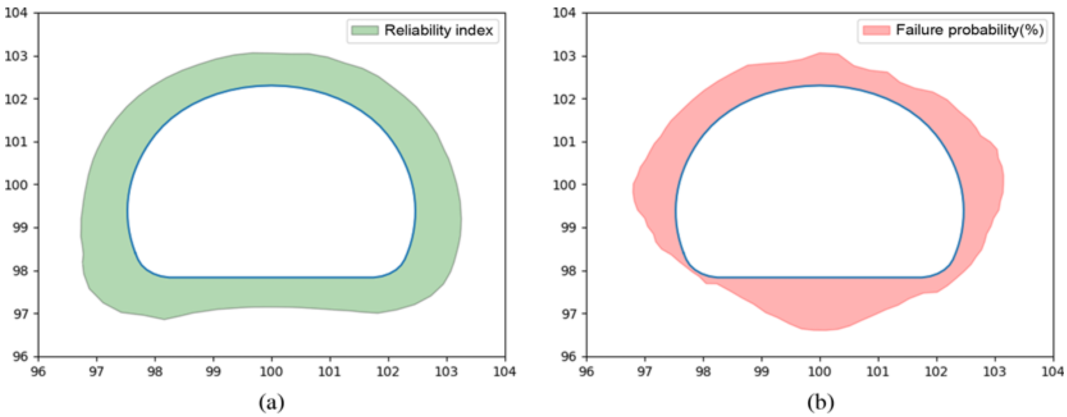

Fig. 15. The Uncertainty of the Surrounding Rock Mass Deformation Induced by Excavation: (a) Reliability Index, (b) Failure Probability

contours of surrounding rock deformation at different design points. The deformation law between them was roughly the same. It also showed that it was feasible to evaluate tunnel uncertainty using the developed method. Fig. 15 exhibits the distribution of reliability index and failure probability along the tunnel wall under the surrounding rock mass deformation criterion. It captured well the deformation and failure behavior of the surrounding rock mass and conformed to rock mechanics theory. It further proved that the SPCE surrogate model could capture the deformation and mechanical behavior of the surrounding rock mass in tunnel excavation instead of the numerical model. This study provided a reliable, scientific, and promising uncertainty evaluation method based on the SPCE surrogate model.

6. Conclusions

A novel uncertainty analysis framework was developed to consider the uncertainty of tunnels by combining the SPCE, FORM, and numerical models. The SPCE model was regarded as a surrogate model to generate the limit state function. FORM was utilized to compute the reliability index based on the SPCE surrogate model. The SHGO was utilized to seek the solution to the optimization problem in FORM. The SPCE-based MCS was used to estimate failure probability. The circular tunnel and horseshoe tunnel were taken as examples to validate the developed method. The SPCE surrogate model developed in this study can replace numerical modeling in the uncertainty analysis for rock engineering. The followings summarize the results and conclusions of this study.

1. The SPCE-based surrogate model characterized the mechanical behavior and deformation law of the rock mass well during tunnel excavation. The SPCE-based surrogate model could approximate well the complex, nonlinear, and implicit limit state function in uncertainty analysis and could enhance the numerical simulation efficiency. In practical rock engineering, numerical modeling is costly for uncertainty analysis and needs repeated computations. The SPCE-based surrogate model offered an effective and promising way to enhance the performance of uncertainty analyses and prediction accuracy.
2. The uncertainty analysis was conducted using the developed framework and MCS, respectively, and they were in good agreement with the existing solutions. The results proved that the developed framework could capture the uncertainty

of rock engineering problems well, and it was feasible to use the SPCE-based surrogate model and reliability method to handle the uncertainty.

3. The performance and accuracy of the uncertainty analysis depend on the numerical model. The selection of an appropriate numerical model is essential for the successful application of the SPCE-based surrogate model.

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