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# A fully coupled thermo-hydro-mechanical model for rock mass under freezing/thawing condition

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ABSTRACT

The coupled thermo-hydro-mechanical (THM) process in rock mass under freezing/thawing condition is studied. The governing equations for THM coupling of freezing/thawing rock are established based on the mass conservation law, energy conservation law and the principle of static equilibrium. The phase change of water in the rock is studied, and the frozen ratio function considering time-dependent behavior is proposed. Finally, we implement the THM coupling equations under freezing/thawing condition into three-dimensional finite-difference program to simulate the cooling test of an underground cavern for gas storage at low temperature. The distribution of temperature and stress fields is monitored and compared with the recorded results from field test, which shows that the simulation result is close to the field test.



## 1. Introduction

Frost weathering of rock mass poses serious threat to the stability of geotechnical engineering in cold regions. Freeze/thaw action is usually most active at some finite depth where the rate of cooling is sufficiently slow for significant moisture to migrate to micro-fractures as the freezing front advances (Hallet et al., 1991). The problem of frost weathering of rock mass involves thermo-hydro-mechanical (THM) coupling at low temperature, which has long been considered as important process in the engineering rock mass. Important contemporary issues, such as geothermal exploitation and the disposal of radioactive wastes, have invoked many research efforts aiming to reveal the coupled THM behavior of geological systems (Li et al., 2013).

The primary distinction of THM coupling process at low temperature and normal temperature is the participation of phase change, which is a complicated process in pores or cracks of the rock mass. The water and heat migration in the freezing rock is a very import process affecting phase change in the rock. Earlier research on the THM coupling process in porous media mainly focused on the water and heat migration in freezing/thawing soil and the driving force is mainly attributed to the capillary effect (Philip and De Veries, 1957). Later, the capillary frost model appeared (Everett, 1961; Penner, 1959). From 1970's, another driving force called segregation potential attracted people's notice. The water migration due to segregation potential at the freezing front of the rock can also induce frost damage (Konrad and Morgenstem, 1982; Miller, 1972). Afterwards, some researchers pointed out that water and heat migration is also affected by temperature gradient (Loch and Kay, 1978; Yang et al., 2006). Present researches of multi-fields coupling have mainly focused on the TH coupling or heat transfer in freezing rock (Lai et al., 2012; Liu et al., 2013; Tan et al., 2011; Zhang et al., 2013), and few fully coupled THM models for rock mass under freezing/thawing conditions were proposed.

Besides water and heat migration, the following problems should be investigated to study the mechanism of THM coupling process in the rock mass under freezing/thawing condition.

(1) The phase change of water in pores or cracks is a special process as the freezing point changes with confining pressure. When water freezes, a 9% volume expansion occurs, inducing freezing pressure if confined by the rock skeleton. Thus, the pressure of unfrozen water would increase as the ice crystal grows. We know that the freezing point of water decreases when pressure increases, therefore, the freezing point of water in pores or cracks might not be constant. This condition will cause great influence when distinguishing freezing and unfrozen zones in numerical analysis.
(2) A key variable called frozen ratio should be investigated in the freezing process. Frozen ratio is defined as the percentage of water phase changed into ice, which is affected by multiple factors such as freezing temperature, freezing time, etc. This variable has close relation with freezing pressure and water-heat migration in freezing fringe.

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In the present work, we derive the governing equations for THM coupling of rock under freezing/thawing condition according to the mass conservation law, energy conservation law and the principle of static equilibrium. Afterwards, the frozen ratio function and effective freezing pressure are obtained based on phase change theory and the energy conservation law. Finally, the cooling test of underground cavern in Röda Sten Rock Laboratory (RSRL) is simulated by implementing the THM coupling equations under freezing/thawing condition into three-dimensional finite-difference program. Deformation of the cavern, as well as distribution of temperature and stress fields is monitored and compared with the recorded results.

## 2. Theoretical study of THM coupling under freezing/thawing condition

### 2.1. Mechanism of THM coupling under freezing/thawing condition

Thermal, hydraulic and mechanical fields in the rock mass would interact with each other. Part of the water would change into ice when temperature drops to subfreezing, leading to a quite different THM coupling mechanism in freezing/thawing rock mass. The coupling mechanism of THM fields under freezing/thawing conditions can be demonstrated as in Fig. 1.

When temperature stays at normal level, the thermal field would affect hydraulic field by thermal convection and be affected by fluid-heat transport in turn. The hydraulic field would influence the mechanical field through pore pressure and be affected by bulk strain. The mechanical field would affect the thermal field through mechanical work and bulk strain and be affected by the thermal strain due to temperature change (Ji et al., 2011). On the other hand, when temperature drops to subfreezing, phase change occurs and the THM coupling mechanism becomes quite different. Firstly, the latent heat would affect the thermal field, and volume expansion of water during freezing process would exert additional pressure (called freezing pressure or ice pressure) to the mechanical field. Furthermore, the freezing pressure as well as the segregation effect of ice crystal would induce a driving force to the hydraulic field. Meanwhile, the permeability of the rock mass would decrease as the ice crystal blocks the cracks besides reducing liquid water. However, if the ice pressure exceeds the tensile strength of the rock skeleton, the cracks will propagate and coalesce. As a result, the permeability coefficient increases. Such is the complicated interactions among thermal, hydraulic and mechanical fields.

### 2.2. The THM coupling mathematical model

The governing equations of THM coupling process in freezing rock would be summarized in this part. In the following research, the freezing/thawing rock is treated as continuum system although the rock mass is actually rich in micro-cracks. Considering that the multi-field coupling process involves unsaturated water migration, heat transport, and elasto-plastic deformation, the following assumptions are made to facilitate the research work:

(1) The rock mass is a multiphase system containing rock grain, liquid water, solid ice and air, and the rock material is homogeneous and isotropic. In addition, the interaction between stress/strain and fluid flow in the freezing/thawing rock is governed by Biot's consolidation theory (Biot, 1941).
(2) The fluid flow satisfies extended Darcy law and the heat flux satisfies Fourier's law, and the coupled process between stress/strain and heat transfer in the rock mass is based on thermo-elastic consolidation theory.
(3) The super-cooling stage of the water is neglected, which means water would freeze once the temperature drops to freezing point. The temperature of ice-water system maintains at freezing point during the freezing/thawing process.
(4) The pressures of liquid water and ice roughly equal to each other in a single pore, while the air is strongly compressible and the gas pressure can be neglected before water/ice fills up the pore. The main purpose of this assumption is to get the effective force due to volume expansion of ice/water system. Besides, the evaporation of air could be ignored. The pressure increment of ice-water system due to freezing/thawing action is defined as freezing pressure.

As shown in Fig. 2, the freezing rock mass is considered to be constituted by rock grain, ice, unfrozen water and air. The following relation can be given.

$$V = V_s + V_w + V_i + V_a \tag{1}$$

$$m = m_s + m_w + m_i + m_a \tag{2}$$

where, $V$ and $m$ is the total volume and mass of the rock mass, the subscripts $s$, $w$, $i$ and $a$ represent solid grain, unfrozen water, ice and air, respectively.

The volume fraction can be defined as $n^\alpha = V^\alpha/V(\alpha = s, w, i, a)$. Then, the porosity can be written as $n = n^w + n^i + n^a$. The frozen ratio that represents the percentage of water changed into ice can be written as:

$$u_i = \frac{m_i}{m_i + m_w} = \frac{\rho_i n^i}{\rho_i n^i + \rho_w n^w}. \tag{3}$$

#### 2.2.1. Mechanical equilibrium equation

The general balance equation can be expressed as:

$$\nabla \cdot \boldsymbol{\sigma} + \rho_m \mathbf{g} = 0 \tag{4}$$

where, $\boldsymbol{\sigma}$ stands for the total stress tensor, $\mathbf{g}$ is gravity acceleration vector and $\rho_m$ is density of the rock mass.

$$\rho_m = nS_r[u_i\rho_i + (1-u_i)\rho_w] + n(1-S_r)\rho_a + (1-n)\rho_s \tag{5}$$

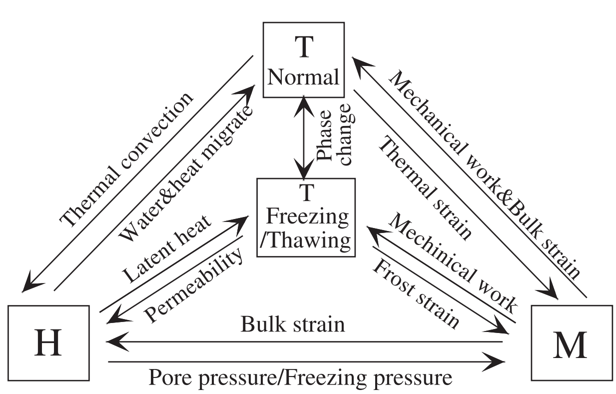



**Fig. 1.** Mechanism of THM coupling under freezing/thawing condition.

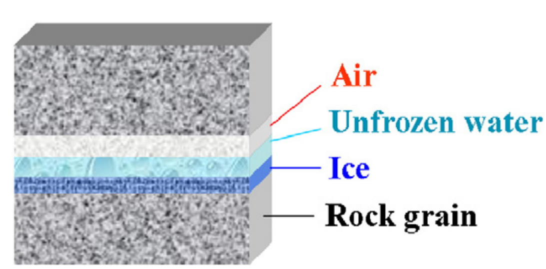



**Fig. 2.** Sketch for constitution of the freezing/thawing rock.

where, $n$ is the porosity, $S_r$ is the saturation degree, $\rho_i$, $\rho_w$, $\rho_a$ and $\rho_s$ are density of ice, water, gas and rock grain respectively.

Thermal strain due to temperature change can be expressed as:

$$\varepsilon_{ij}^{T} = \beta \Delta T \delta_{ij} \tag{6}$$

where, $\beta$ is the coefficient of linear thermal expansion of rock mass, $\Delta T$ denotes temperature change and $\delta_{ij}$ is the Kronecker delta.

Considering the unsaturated condition, the mass balance equation of the freezing/thawing rock mass can be expressed as:

$$\nabla \cdot \left[\boldsymbol{\sigma}^{e} - (\alpha_{wi} P_{wi})\boldsymbol{I}\right] - \nabla \cdot E(\beta \Delta T)\boldsymbol{I} + \rho_m \mathbf{g} = 0 \tag{7}$$

where, $\boldsymbol{\sigma}^{\mathbf{e}}$ is the effective stress tensor; $P$ denotes pressure and the subscripts $w$, $i$, $a$ represent liquid water, ice and air respectively; $E$ is the young's modulus; $\mathbf{I}$ the identity tensor; $\alpha_{wi}$ is the incremental effective stress parameter for ice-water system (Chen et al., 2009).

In particular, the general mechanical constitutive relations for elastic state in small strain can be expressed as:

$$\boldsymbol{\sigma} + \kappa \alpha_{wi} P_{wi} \boldsymbol{I} = 2G(\boldsymbol{\varepsilon} - \beta \Delta T \boldsymbol{I}) + \left(K - \frac{2}{3}G\right) \cdot \left(I'_1 - 3\beta \Delta T\right)\boldsymbol{I} \tag{8}$$

where, $\sigma$ and $\varepsilon$ are total stress tensor and total strain tensor respectively, $\kappa$ is frost pressure transmitting coefficient similar to Biot coefficient, $K$ and $G$ are bulk modulus and shear modulus respectively. $I_1'$ is the first invariant of strain tensor, $I'_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \varepsilon_V, \varepsilon_1, \varepsilon_2$ and $\varepsilon_3$ are principal strains, and $\varepsilon_V$ is the volume strain. When the freezing pressure or confining pressure is quite high, plastic strain might occur. The plastic strain is an uncertain factor as it depends on selected yield criteria and flow rules. Using the associated flow rule to determine plastic strain increment as follow:

$$\varepsilon_{ij}^{p} = \lambda^* \frac{\partial f}{\partial \sigma_{ij}} \tag{9}$$

$$\sigma_{ij} = \sigma_{ij}^{e} + P_{wi} \delta_{ij} \tag{10}$$

where, $\lambda^*$ is a non-negative scalar relating to freeze-thaw history, and $f$ is the yield function. $\sigma_{ij}$ and $\sigma_{ij}^{e}$ are components of total stress tensor and effective stress tensor respectively.

### 2.2.2. Mass conservation equation of water

Neglecting the evaporation process of water, the mass conservation equation of water can be expressed as:

$$\frac{\partial[\rho_w n(1-u_i)]}{\partial t} + \frac{\partial(\rho_i n u_i)}{\partial t} + \rho_w \nabla \cdot \mathbf{q}_w = 0 \tag{11}$$

where, $n$ is the porosity; $u_i$ denotes frozen ratio; $\rho$ stands for density. The subscripts $i$ and $w$ represent ice and water respectively. $\mathbf{q}_w$ is the water flux vector. Assuming water fluid satisfies extended Darcy law for unsaturated flow (Bear and Bachmat, 1991), $\mathbf{q}_w$ can be expressed as:

$$\mathbf{q}_w = -n S_r \rho_w \frac{k_{rel}}{\mu} \mathbf{k}(\nabla P - \phi_T \nabla T - \rho_w \mathbf{g}) \tag{12}$$

where, $\mathbf{k}$ is the fluid permeability tensor when the rock mass is saturated, $k_{rel}$ is the relative permeability coefficient, $\nabla P$ is the freezing pressure gradient; $\rho_w$ is the density of water, and $\mathbf{g}$ is the gravity acceleration vector. $\phi_T$ denotes the segregation potential, which is defined as the ratio between migration of water and the temperature gradient at the freezing fringe, and it's considered as one of the important driving forces of water migration; $\nabla T$ is the temperature gradient.

### 2.2.3. Heat transport and energy conservation equation

We assume that the heat flux satisfies Fourier's law. Considering the diffusive and convective transport, the heat flux can be expressed as (Hemmatia et al., 2012):

$$q_T = -\lambda_m \nabla T + n S_r \rho_w C_w \mathbf{q}_w \frac{\partial T_w}{\partial t} \tag{13}$$

where, $\mathbf{q}_w$ is the water flux vector, $C_w$ is the specific heat of water, $\lambda_m$ is the heat conductivity of the rock mass [kJ/(m·s·K)], $\nabla T$ is the temperature gradient, and $T_w$ is the temperature of water.

$$\lambda_m = n S_r \lambda_w + n(1-S_r)\lambda_a + (1-n)\lambda_s \tag{14}$$

where, $\lambda_w$, $\lambda_g$ and $\lambda_a$ are heat conductivity of water, air and solid respectively.

The released or absorbed latent heat of the water/ice system can be expressed as:

$$q^{L} = L\rho_i \frac{\partial n^{i}}{\partial t} = L\rho_i n S_r \frac{\partial u_i}{\partial t} \tag{15}$$

where, $L$ is latent heat coefficient (J/kg).

The energy conservation equation of unsaturated rock mass considering both convection and conduction can be expressed as:

$$C_V \frac{\partial T}{\partial t} + \nabla \cdot (-\lambda_m \nabla T) + L\rho_i \frac{\partial u_i}{\partial t} + C_w \rho_w \frac{\partial T}{\partial t} (\nabla \cdot \mathbf{q}_w) = 0 \tag{16}$$

where, $T$ is temperature of the rock mass [K]; $t$ is time [sec], $C_V$ is the effective bulk specific heat of the rock mass [kJ/(m$^3$·K)], which can be expressed as:

$$C_V = n^s C_s + n^w C_w + n^i C_i = n^s C_s + n S_r (1-u_i) C_w + n S_r u_i C_i \tag{17}$$

$$\rho_m = n^s \rho_s + n^w \rho_w + n^i \rho_i = n^s \rho_s + n S_r (1-u_i)\rho_w + n S_r u_i \rho_i \tag{18}$$

where, $n^s$, $n^w$, $n^i$ denote the volume fraction of rock grain, liquid water and ice, respectively. $C_s$, $C_w$ and $C_i$ are the bulk specific heat of rock grain, liquid water and ice, respectively; $u_i$ is frozen ratio.

Substituting Eqs. (12) and (13) into Eq. (16), we can get the governing equation as:

$$\nabla \cdot \left\{ -\lambda_m \nabla T + \left[ -n^2 S_r^2 \rho_w^2 C_w \frac{k_{rel}}{\mu} \mathrm{k}(\nabla P_{wi} - \rho_w \mathrm{g}) \right] \frac{\partial T_w}{\partial t} \right\} + C_m \rho_m \frac{\partial T}{\partial t} + L\rho_i \frac{\partial u_i}{\partial t} = 0. \tag{19}$$

In order to facilitate the research work, some simplifications and assumptions were made to study the THM coupling of rock mass under freezing/thawing conditions in this paper. The rock is considered as hard and compact porous media with comparatively lower porosity in this model. Thus, the consolidation effect is not considered in this THM model. Actually, the rock mass is usually rich in micro-cracks or flaws. If the more complex inner structure is considered, the governing equations of THM coupling should be much more complicated.

## 3. Study of freezing point based on Clapeyron equation

When the ratio between ice and water roughly achieves a constant value under certain condition, the state can be called equilibrium state. We assume *Clapeyron equation* applies to the equilibrium state in freezing/thawing rock.

$$\frac{dP}{dT_0} = \frac{\Delta H}{T_0 \Delta V}. \tag{20}$$

Eq. (20) is the *Clapeyron equation*. Where, $P$ is the pore pressure, $T_0$ is the corresponding freezing point of water, and $\Delta H$ and $\Delta V$ are increment of enthalpy and volume increment respectively. Specially, for the freezing/thawing water (Fu et al., 2005),

$$dP = \frac{\Delta_{fus} H_m}{\Delta_{fus} V_m T_0} dT_0 \tag{21}$$

where, $\Delta_{fus} H_m$ is the melting mole enthalpy increment of ice, and $\Delta_{fus} V_m$ is the melting mole volume increment of ice.

Take integral for both sides of Eq. (21).

$$P = \frac{\Delta_{fus} H_m}{\Delta_{fus} V_m} \ln T_0 + C_0 \tag{22}$$

$$T_0(P) = k \exp\left(\frac{\Delta_{fus} V_m}{\Delta_{fus} H_m} P\right). \tag{23}$$

We assume that the densities of water and ice are respectively $\rho_w = 999.9$ kg/m$^3$ and $\rho_i = 916.8$ kg/m$^3$ under standard state ($P = 1.01325 \times 10^5$ Pa, $T_0 = 273.2$ K), and $\Delta_{fus} H_m = 333.5$ kJ/kg. Then, the volume expansion of water per kilogram after phase change can be expressed as:

$$\begin{aligned} \Delta_{fus} V &= \frac{1}{999.9\,\text{kg}\cdot\text{m}^{-3}} - \frac{1}{916.8\,\text{kg}\cdot\text{m}^{-3}}. \\ &= -9.06 \times 10^{-5}\text{m}^3\cdot\text{kg}^{-1} \end{aligned} \tag{24}$$

Thus, the expansion coefficient of water when frozen into ice can be derived as:

$$\eta = -\rho_w \Delta_{fus} V \approx 9\%. \tag{25}$$

Therefore,

$$T_0(P) = k e^{-2.72 \times 10^{-10} P}. \tag{26}$$

The freezing point of water under standard atmospheric pressure is 273.15 K, submitting in Eq. (26), we get $k = 273.16$. Using *Matlab*, the relationship between freezing point and pressure (0.1 MPa–20 MPa) is obtained as in Fig. 3. We can see that the curve is quite close to a straight line.

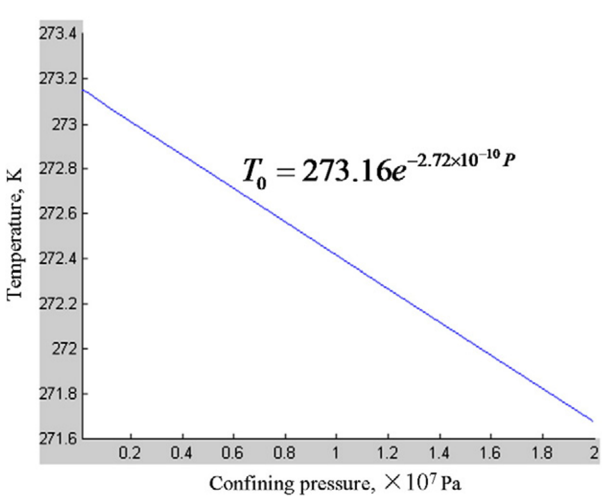



**Fig. 3.** Relationship between freezing point and confining pressure.

## 4. Study of effective freezing pressure

The freezing process under a constant subfreezing temperature is shown as in Fig. 4. Neglecting the water leakage in the pore, the frozen ratio would increase gradually with freezing time, leading an increasing pore pressure. As shown in Fig. 4, the initial pressure, frozen ratio and temperature of the system are equal to $P_0$, $u_0$ and $T_0(P_0)$ respectively. $T_0(P_0)$ is the freezing point under the confining pressure $P_0$. As the freezing action is going on, ice crystal grows and expands, leading to increasing of pressure of the system. Meanwhile, the frozen ratio increases. The latent heat of the system is releasing and transferred out of the water/ice system gradually.

In order to derive the frozen ratio under a certain freezing/thawing condition (temperature, time, etc.), the following assumptions are made.

(1) The increasing of freezing pressure is a quasi-static process. The pores are considered to distribute uniformly in the rock, and the rock element is isotropic.
(2) The latent heat released/absorbed by water/ice in the freezing/thawing process is assumed to completely come from the ice-water system.
(3) The temperature of ice-water system maintains at freezing point during the freezing/thawing process.

Then, the freezing pressure $P_i$ in the pores or micro-cracks can be equivalent to be a hydrostatic pressure $P_e$ loading on the rock element (shown as in Fig. 5).

$$P_e = \kappa P_i \tag{27}$$

where, $\kappa$ is frost pressure transmitting coefficient.

The volume increment of ice-water system without confining pressure can be written as follow:

$$n^i = \eta n S_r u_i \tag{28}$$

$$\Delta_n = [\beta n S_r u_i - n(1-S_r)] H[u_i - \chi] \tag{29}$$

$$H(u_i - \chi) = \begin{cases} 0 & u_i \le \chi \\ 1 & u_i > \chi \end{cases} \tag{30}$$

$$\chi = \frac{1-S_r}{\beta S_r} \tag{31}$$

where $n^i$ is the volume fraction of ice; $\Delta n$ is the increment of porosity neglecting bulk strain, $u_i$ is the frozen ratio function. It's a key variable quantity controlling the frozen effect, and it is determined by multiple

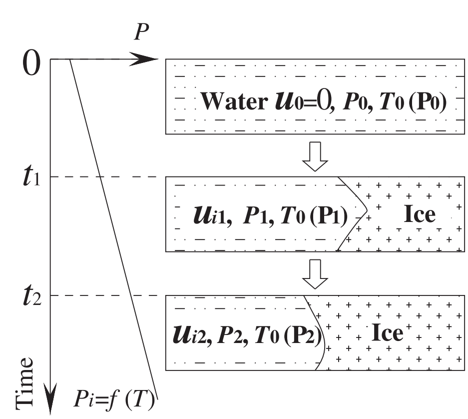



**Fig. 4.** Freezing process under a constant subfreezing temperature.

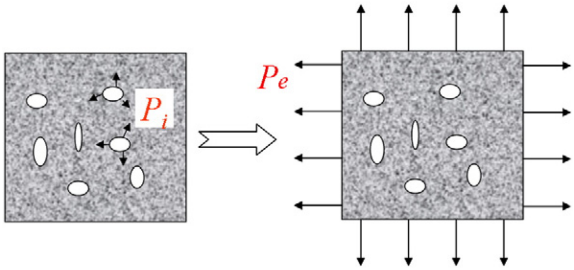



**Fig. 5.** Effective freezing pressure.

factors such as temperature, pressure and frozen time. $H$ is a *Heaviside* function. As ice crystal grows in the pores, unfrozen water will transport to unsaturated zone, and there is no freezing pressure before $u_i$ exceeds the critical value of $(1 - S_r)/\beta S_r$. After that, the subsequent expansion of ice would cause a frost pressure on the rock skeleton. Although part of unfrozen water might migrate to outside of the rock element, the influence is neglected.

Considering the confining effect of the rock skeleton, the bulk strain of rock skeleton and ice can be expressed as:

$$\varepsilon_{VS} = \frac{P_e}{K_s} \tag{32}$$

$$\varepsilon_{Vi} = \frac{P_i}{K_i} = \frac{P_e}{nK_i}. \tag{33}$$

The following relations can be derived.

$$\varepsilon_{VS} + nS_r\varepsilon_{Vi} = [\beta nS_r u_i - n(1-S_r)]H(u_i-\chi). \tag{34}$$

Thus,

$$\left[\frac{1}{K_s} + \frac{S_r}{K_i}\right]P_e = [\beta nS_r u_i - n(1-S_r)]H(u_i-\chi). \tag{35}$$

The equivalent freezing pressure can be written as:

$$P_e = \frac{[\beta nS_r u_i - n(1-S_r)]K_sK_i}{K_sS_r + K_i}H(u_i-\chi). \tag{36}$$

Specifically, when the rock is initially saturated, $S_r = 1$ and $H(u_i - \chi) = 1$. Thus,

$$P_e = \frac{\beta nS_r u_i K_s K_i}{K_sS_r + K_i}. \tag{37}$$

## 5. Study of frozen ratio in thermal insulation system

The latent heat released/absorbed by water/ice in the freezing/thawing process completely comes from the ice-water system. Thus, the system constituted by rock grain, water and ice can be considered as thermal insulation system. The heat conduct between water and ice obeys Fourier's law. Neglecting the volume expansion work of water/ice in phase change process, the following relation can be obtained.

$$Ln\rho_w S_r \frac{\partial u_i}{\partial t} = \lambda_{wi}(T_0 - T_s) \tag{38}$$

where, $t$ is time; $T_0$ is the freezing point, $T_s$ is the temperature of rock grain. $\lambda_{wi}$, is heat conductivity of ice-water system, which can be assigned to be a constant value roughly equals to $\lambda_w$, the heat conductivity of water.

Solving the partial differential equation, we get:

$$t = -\frac{(1-n)C_s\rho_s}{\lambda_{wi}}\ln(T_0 - T_s) + C_1 \tag{39}$$

and,

$$T_s = T_0 - C_2 e^{-\frac{\lambda_{wi}t}{(1-n)C_s\rho_s}} \tag{40}$$

where, $C_1$, $C_2$ are both constant.

Considering temperature boundary condition, when $t = 0$, $T_s = T_{s0}$ and $C_2 = T_0 – T_{s0}$.

$$T_0 - T_s = (T_0 - T_{s0})e^{-\frac{\lambda_{wi}t}{(1-n)C_s\rho_s}}. \tag{41}$$

Thus, according to Eqs. (38) and (41), we have:

$$\frac{\partial u_i}{\partial t} = \frac{\lambda_{wi}}{Ln\rho_w S_r}(T_0 - T_{s0})e^{-\frac{\lambda_{wi}t}{(1-n)C_s\rho_s}}. \tag{42}$$

Assuming $\frac{\partial u_i}{\partial t} = Ae^{Bt}$, we get:

$$A = \frac{\lambda_{wi}}{Ln\rho_w S_r}(T_0 - T_{s0}), \quad B = -\frac{\lambda_{wi}}{(1-n)C_s\rho_s}. \tag{43}$$

Therefore,

$$u_i = \int_t Ae^{Bt}dt = \frac{A}{B}e^{Bt} + C_3 \tag{44}$$

where, $C_3$ is constant.

Boundary condition is assigned as: when $t = 0$, $T_s = T_{s0}$, $T_{wi} = T_0$, $u_i = 0$.We get: $C_3 = -\frac{A}{B} = \frac{(1-n)C_s\rho_s}{Ln\rho_w S_r}(T_0 - T_{s0})$..Therefore,

$$u_i(t) = \frac{(1-n)C_s\rho_s}{Ln\rho_w S_r}(T_0 - T_{s0})\left(1 - e^{-\frac{\lambda_{wi}t}{(1-n)C_s\rho_s}}\right). \tag{45}$$

There are two types of heat transfer in the thermal insulation system (constituted by rock grain and ice-water system), including heat conduction and latent heat. According to the second law of thermodynamics, heat can only transfer from hot part to the cold part of the object. Thus, the temperature of rock grain will be lower than the ice-water system all through the freezing process and be higher all through the thawing process in the thermal insulation system. Considering the following ideal condition: the latent heat released by water just makes the temperature of rock grain rise to the freezing point, the following relation can be given:

$$Ln\rho_w S_r = C_s\rho_s(1-n)\Delta T_{se}. \tag{46}$$

Then,

$$\Delta T_{se} = \frac{Ln\rho_w S_r}{C_s\rho_s(1-n)} \tag{47}$$

where, $\Delta T_{se}$ can be called critical temperature change. The following relation can be derived:

(1) $T_0 – T_{s0} = \Delta T_{se}$: when temperature of rock grain rise to $T_0$ (the freezing point), the water is just right completely frozen ($u_i = 1$).
(2) $T_0 – T_{s0} < \Delta T_{se}$: when temperature of rock grain rise to $T_0$, the water cannot completely be frozen ($u_i < 1$). That's because the initial temperature of rock grain is not cold enough, and frozen ratio cannot reach to 1.

(3) $T_0–T_{s0} > \Delta T_{se}$: The rock grain is very cold, and the temperature is still lower than $T_0$ when the water is completely frozen, as the freezing process is going on, the temperature of the ice drops rather than maintains at the freezing point.

We can easily derive the effective range of Eqs. (39)–(41) as follow:

(1) when $T_0–T_{se} \le \Delta T_{se}$, $0 \le t < +\infty$.
(2) when $T_0–T_{se} > \Delta T_{se}$, $0 \le t < t_e$.

$$t_e = \frac{(1-n)C_s\rho_s}{\lambda_{wi}} \ln \frac{T_0-\Delta T_{se}}{T_0-T_{s0}-\Delta T_{se}} \tag{48}$$

where, $t_e$ is the time when water is frozen completely when $T_0–T_{se} > \Delta T_{se}$.

Thus, the expression of frozen ratio can be written as follow:

(1) when $T_0–T_{s0} \le \Delta T_{se}$

$$u_i(t) = \frac{(1-n)C_s\rho_s}{Ln\rho_w S_r}(T_0-T_{s0})\left(1-e^{-\frac{\lambda_{wi}t}{(1-n)C_s\rho_s}}\right) \quad (0 \le t < +\infty) \tag{49}$$

(2) when $T_0–T_{s0} > \Delta T_{se}$ and $0 \le t < t_e$

$$u_i(t) = \begin{cases} \frac{(1-n)C_s\rho_s}{Ln\rho_w S_r}(T_0-T_{s0})\left(1-e^{-\frac{\lambda_{wi}t}{(1-n)C_s\rho_s}}\right) & (0 \le t < t_e) \\ 1 & (t \ge t_e) \end{cases} \tag{50}$$

$$t_e = B \ln \frac{A}{A-1} \tag{51}$$

$$B = \frac{(1-n)C_s\rho_s}{\lambda_{wi}}, A = \frac{(1-n)C_s\rho_s}{Ln\rho_w S_r}(T_0-T_s). \tag{52}$$

Eqs. (49) to (52) interpret the relation between frozen ratio and time under thermal insulation condition, which can be described as in Fig. 6.

## 6. Engineering application and validation study

This paper tries to simulate the cooling test of underground cavern initiated by R. Glamheden and Lindblom in 2002. THM coupling process is to be simulated during the cooling process using FLAC3D. The THM coupling equations under freezing/thawing conditions have been implemented into finite-difference program. Afterwards, the temperature and deformation of the inner wall of the cavern are monitored and compared with the recorded data from Glamheden's test.

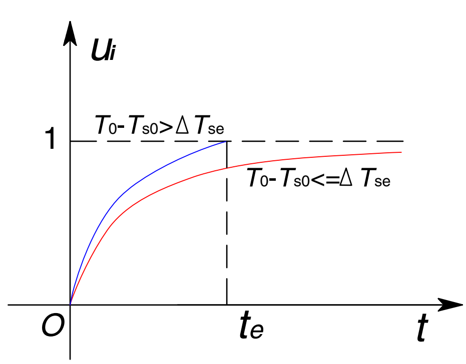



**Fig. 6.** Relation between frozen ratio and freezing time under thermal insulation condition. $T_0$ is the freezing point; $T_{s0}$ is the initial temperature of the rock grain; $t_e$ is the time when water frozen completely when $T_0–T_{se} > \Delta T_{se}$; $\Delta T_{se}$ is the critical temperature change.

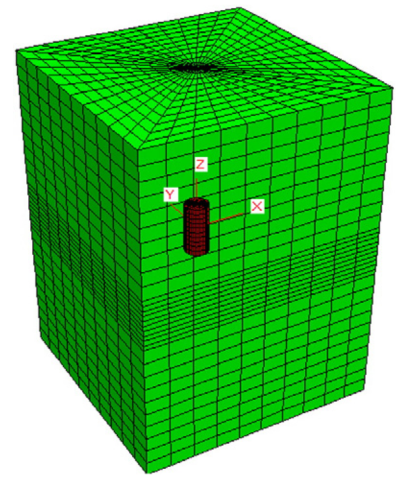



**Fig. 7.** Calculating model.

### 6.1. Engineering introduction

Röda Sten Rock Laboratory (RSRL) is a facility in Gothenburg, Sweden, operated by Chalmers University of Technology. This site was commissioned mainly for large-scale testing of technology for underground gas storage. The testing cavern is located approximately 70 m below ground level and approximately 30 m below the water table of the nearby river. The test cavern is constructed as a vertical cylinder with a diameter of 7 ± 0.5 m and a height of 15 ± 0.5 m. The dominating rock type around cavern is a granodiorite with gneiss structure and elements of alkali granite (Glamheden and Lindblom, 2002).

The initial ambient temperature in the rock mass was approximately 10 °C. Firstly, the cavern temperature was decreased to 0 °C and held constant at this level for approximately 80 days. The temperature was decreased in four steps of 10 °C from 0 °C to −40 °C. Each step lasted for 2 weeks. On level −40 °C, the temperature was maintained constant for 40 days, provided through a tunnel. The maximum frozen depth in the side wall is about 6.75 m, and the maximum radial displacement of 10.5 mm occurs on mid-height of the cavern near one of the interfaces.

### 6.2. Simulating program

FLAC3D is proficient at simulation involving THM coupling problems. But, the built-in coupled THM calculating method doesn't contain

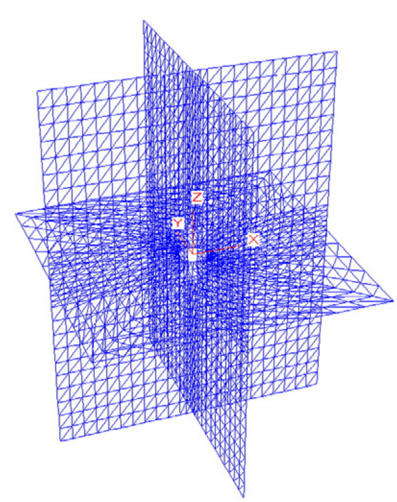



**Fig. 8.** Interfaces.

**Table 1**
Main parameters for the calculating model.

| Parameters | Value |
|---|---|
| Rock mass Young's modulus (GPa) | 42 |
| Rock Poisson's ratio | 0.23 |
| Compressive strength (MPa) | 20 |
| Cohesion (MPa) | 4.5 |
| Friction angle (°) | 40 |
| Dilation angle (°) | 10 |
| Coefficient of thermal expansion ($10^{-6}$/°C) | 4.28 |
| Thermal diffusivity ($10^{-6}$ $m^2$/s) | 1.65 |
| Thermal conductivity (W/m·°C) | 2.67 |
| Specific heat (J/kg·°C) | 602.3 |
| Density (kg/$m^3$) | 2689 |
| Interface normal stiffness (GPa/m) | 76 |
| Interface shear stiffness (GPa/m) | 38 |
| Interface dilation angle (°) | 36.5 |
| Interface friction angle (°) | 6 |

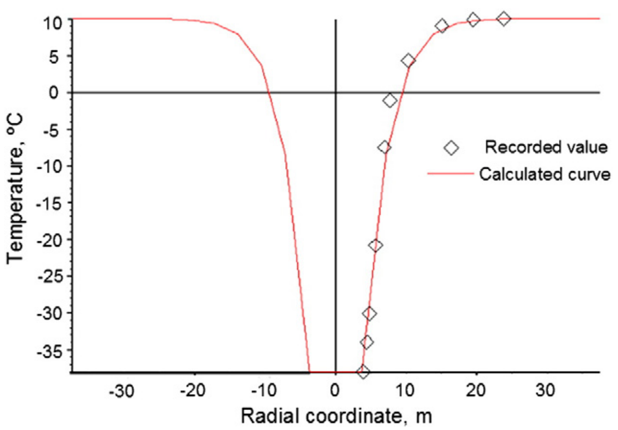



**Fig. 9.** Calculated and recorded temperature in the rock mass during cooling at the cavern mid-height.

the phase change process in the rock. THM coupling equations under freeze-thaw condition, as well as the freezing pressure could be implemented by FISH programs in calculating procedure.

The calculating model with the size of 100 × 75 × 75 m is established using FLAC3D, shown as in Fig. 7. The cavern (15.5 m in height and 7.5 m in diameter) is located in the center in the model. As shown in Fig. 8, a group of interfaces were built in the model according to the description of the geological characteristic in the reference.

The mechanical and thermal parameters in the calculating process are listed in Table 1 (Glamheden and Lindblom, 2002).

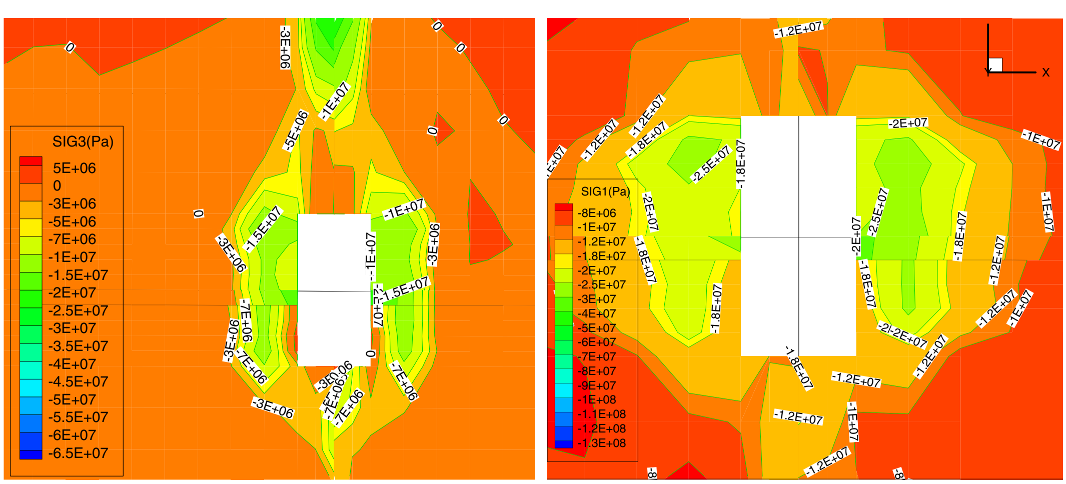
The cavern is 30 m below the water table of the nearby river. Thus, the water table was set on the plane that is 30 m above the cavern top. And, the rock below water table is set initially saturated. The initial value of porosity and seepage coefficient of the rock mass is set to 1.5% and $5.0 \times 10^{-19}$ ($m^2$/Pa·s).

The model was self mechanically balanced before excavation, and the displacement of grid points was initialized to be zero before cooling. The cooling process is strictly implemented according to field test described in Part 6.1. The initial freezing point is set to −2 °C, and it's modified according to local pore pressure. The additional freezing pressure at each grid is modified every 2 h by FISH program according to the element temperature.

The calculated and recorded temperature in the rock mass during cooling at the cavern mid-height is shown as in Fig. 9. We can see that, the calculated result is quite close to the recorded values from Glamheden's test.

The stress field on the plane of $y = 0$ is shown in Fig. 10. We can see that freezing action is active at a depth of approximately 7 m until the end of the cooling process. Both of the maximum and minimum principal stress rise and then decrease with the depth in the wall. The maximum principal stress exceeds 20 MPa under ground stress and freezing pressure. The influence of freezing action on the stress field is obvious. It can be inferred from Figs. 9 and 10 that the frozen depth in side wall is approximately 6.5 m, while the tested value by Glamheden is 6.75 m. Thus, the calculated result roughly coincides with the recorded value. We can conclude that the freezing/thawing action is generally active within finite depth of the rock mass. Besides thermal insulation measures and lowering water content, reinforcement of this region is necessary to prevent frost damage.

The radial displacement of the cavern mid-height was show as in Fig. 11. The cavern temperature is firstly decreased to 0 °C and maintains constant at this level for approximately 80 days. This period is considered as preparing stage, and phase change has not occurred. The rock mass behaves as thermal contraction characteristic as the decrease of temperature. Thus, an initial displacement of 12 mm appears. Shown as in Fig. 11, when the freezing temperature drops to −10 °C, water in the rock starts freezing and frost expansion occurs. The cavern




**(a)** Minimum principal stress **(b)** Maximum principal stress

**Fig. 10.** Distribution of stress field on the plane of $y = 0$.

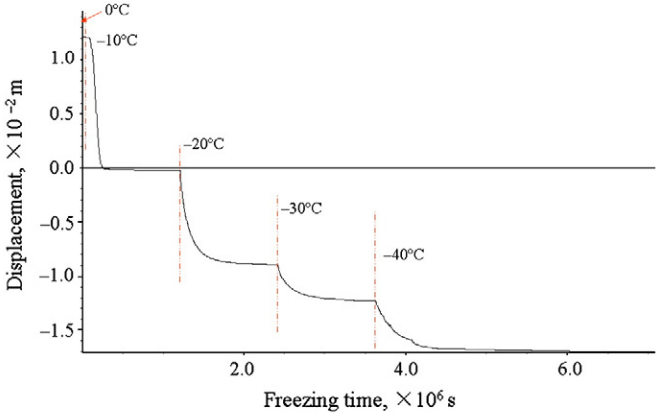



**Fig. 11.** Displacement of inner face at the cavern mid-height.

shrinks gradually as cooling step proceeds. The final displacement is approximately 17 mm.

## 7. Conclusions

(1) Based on the mass conservation law, energy conservation law and the principle of static equilibrium, the thermo-hydro-mechanical coupling mechanism is studied, and a new THM coupling model considering phase change effect is established. The water migration caused by segregation potential and temperature gradient is described in the THM coupling equations.
(2) The phase change of water in the rock is studied, and the relation between freezing point and pore pressure (0.1 MPa–20 MPa) is derived according to Clapeyron equation. The freezing point is demonstrated to decrease as pressure is increasing, and the relation curve is quite close to a straight line.
(3) Frozen ratio, which represents the percentage of water changed into ice, is time-dependent and related to freezing temperature. The frozen ratio with the form of exponential function is proposed based on the phase change theory and the thermodynamics law.
(4) The cooling test of gas storage cavern initiated by R. Glamheden in Röda Sten Rock Laboratory is simulated. The THM coupling process is simulated. Distribution of temperature and stress fields is monitored and compared with the recorded results from field test, proving that the calculated result is satisfying.

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