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# Dynamic response of subgrade under action of vehicle load considering pavement material roughness

# J. Liu\*, H. L. Yao, P. Chen and Z. Lu

Pavement roughness in the highway greatly affects the performance of the subgrade structure. In this paper, a vehicles, roughness pavement, layered subgrade coupled dynamics analysis model is built considering the roughness of the pavement material. The roughness waveform of the pavement material is simplified to be a sine wave, and the vehicle is represented by two degrees of freedom. One of fourth vehicle model is used in the subgrade model. The vehicle load varying with the time is obtained using the vehicle model with consideration of the pavement roughness, which is applied in the numerical model. Finally, subgrade dynamic response and deformation values under the action of multiple cyclic vehicle loads are obtained. The comparison of pavement deformation between coupling calculation and non-coupling calculation is discussed. The study results show that, the deformation of the subgrade and pavement is related to the amplitude and wavelength of pavement roughness. In the interaction of the sunken position of the pavement and the vehicle load, a large deformation is produced. The variations of the pavement deformation are the same in different depths of the subgrade. It is also shown that the deformation of the pavement in coupling calculation and non-coupling calculation is different. The results suggest that it should be more realistic with consideration of the roughness of the pavement material.

Keywords: Subgrade, Roughness, Pavement material, Dynamic Response, Vehicle load deformation

# Introduction

It is of great importance to investigate on reducing the impact of the traffic pressure since the increasing traffic pressure could cause many problems in subgrade structure performance, operating costs, service life.<sup>1,2</sup> The impact of the vehicle to the road surface typically includes vehicle structure parameters, the speed of the vehicle, the weight and mass distribution. Pavement roughness has been shown to be the main cause of the vehicle vibration 3-5.3-5 The vibration caused by the vehicle factor itself is smaller than that by the road roughness. The dynamic load is determined by the roughness of the road surface. For instance, the pavement roughness could greatly accelerate the pavement destruction. However, the pavement and structure are typically separated in most studies<sup>6,7</sup> of subgrade structure performance evolution targeting subgrade under vehicle loads, and the pavement roughness is studied by simple manners in these studies. As mentioned early, the structural performance of subgrade and the life of the vehicle are affected by mutual effects of pavement and vehicle load. Therefore, in this study, the structure of the vehicle and the pavement coupling effect is investigated. Furthermore, roughness waveform expression of pavement is introduced, the theoretical expression of vehicle loads is developed, which is applied into the subgrade numerical model. From iterative calculation, the dynamic response of pavement structural under vehicle loads is analysed, vehicles, roughness pavement layered subgrade coupled dynamics analysis model is built with consideration of the roughness of the pavement material.

## Theoretical analysis

#### Pavement roughness

The tendency of longitudinal roughness of pavement is seemed to be superposition of sinusoidal wave with different amplitude. The pavement roughness is written  $by^8$ 

$$y_0(t) = h_0 \sin(\omega t) \tag{1}$$

where  $h_0$  is the amplitude of rough pavement,  $\omega = 2\pi V/\lambda$  is the vehicle speed,  $\lambda$  is irregularity wavelength of pavement, *t* is time.  $h_0$  can be considered to be the maximum value in the recessed and arched area of pavement.

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(7)

(8)

(10)

 $\overline{z}_2 = e^{-\left(\frac{c_2}{m_2} + \frac{c_2}{m_1}\right)t} \left(M \cos\left\{\left[\frac{k_2}{m_1} + \frac{k_2}{m_2} - \left(\frac{c_2}{m_2} + \frac{c_2}{m_1}\right)^2\right]^{1/2}t\right\}\right)$ 

 $+\left\{N\sin\left\{\left[\frac{k_{2}}{m_{1}}+\frac{k_{2}}{m_{2}}-\left(\frac{c_{2}}{m_{2}}+\frac{c_{2}}{m_{1}}\right)^{2}\right]^{1/2}t\right\}\right\}\right\}$ 

 $Q = \left[ \frac{G^2 + H^2}{\left(\frac{k_2}{m_1} + \frac{k_2}{m_2} - \omega^2\right)^2 + 4\left(\frac{c_2}{m_2} + \frac{c_2}{m_2}\right)^2 \omega^2} \right]^{\frac{1}{2}}$ 

 $\phi = \arcsin\left[\frac{\left(\frac{c_2}{m_2} + \frac{c_2}{m_1}\right)\omega G - \left(\frac{k_2}{m_1} + \frac{k_2}{m_2} - \omega^2\right)H}{\left(\frac{k_2}{m_1} + \frac{k_2}{m_2} - \omega^2\right)^2 + 4\left(\frac{c_2}{m_2} + \frac{c_2}{m_2}\right)^2\omega^2}\right]$ 

 $z_2 = Q \sin(\omega t - \phi), \quad F(t) = k_1 z_1 + c_1 z'_1$ 

Additional vehicle dynamic load is

 $z_2^* = Q \sin(\omega t - \phi)$ 

 $z_2$  can be written as

where M and N are constant

#### Vehicle model and dynamic load

Vertical vibration acceleration of the vehicle is reduced to be two degrees of freedom vehicle vibration model of 1/4 vehicle body structure in the vehicle numerical model.<sup>9</sup> It has been shown to be able to provide satisfactory accuracy. The forced vibration differential equation of two freedom degrees system of vehicle model can be written as<sup>10</sup>

$$m_{1}\ddot{y}_{1} + c_{1}(\dot{y}_{1} - \dot{y}_{0}) + k_{1}(y_{1} - y_{0}) - c_{2}(\dot{y}_{2} - \dot{y}_{1}) - k_{2}(y_{2} - y_{1}) = 0$$

$$m_{2}\ddot{y}_{2} + c_{2}(\dot{y}_{2} - \dot{y}_{1}) + k_{2}(y_{2} - y_{1}) = 0$$

$$(2)$$

Assuming  $z_1 = y_1 - y_0$ ,  $z_2 = y_2 - y_1$ , the formula (2) can be written as

$$\ddot{z}_{1} + \frac{c_{1}}{m_{1}}\dot{z}_{1} + \frac{k_{1}}{m_{1}}z_{1} - \frac{c_{2}}{m_{1}}\dot{z}_{2} - \frac{k_{2}}{m_{1}}z_{2} = -\ddot{y}_{0},$$

$$\ddot{z}_{1} + \dot{z}_{2} + \frac{c_{2}}{m_{2}}\dot{z}_{2} + \frac{k_{2}}{m_{2}}z_{2} = -\ddot{y}_{0}$$
(3)

Both sides of above equation can be rearranged as

$$\frac{c_1}{m_1}\dot{z}_1 + \frac{k_1}{m_1}\ddot{z}_1 = z_2 + \left(\frac{c_2}{m_2} + \frac{c_2}{m_1}\right)\dot{z}_2 + \left(\frac{k_2}{m_1} + \frac{k_2}{m_2}\right)z_2 \quad (4)$$

 $z_1 =$ 

$$z_{1} = Z \sin(\omega t + \theta) = \sin(\omega t + \theta) \\ = \frac{\left[\left(\frac{c_{1}c_{2}^{2}w^{5}h_{0}}{m_{1}m_{2}}\left(\frac{1}{m_{1}} + \frac{1}{m_{2}}\right)\right]^{2} + \left[h_{0}w^{2}\left(\frac{k_{1}}{m_{1}} - w^{2}\right)\left(\frac{k_{2}}{m_{2}} - w^{2}\right)^{2} - \frac{k_{2}^{2}}{m_{1}^{2}}h_{0}w^{4} + \frac{k_{2}}{m_{1}}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)h_{0}w^{2}\left(\frac{k_{1}}{m_{1}} - w^{2}\right) - \frac{k_{2}}{m_{1}}h_{0}w^{4}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)\right]^{2}}{\left[\left(\frac{k_{1}}{m_{1}} - w^{2}\right)^{2}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)^{2} + \frac{k_{2}^{2}}{m_{1}^{2}}w^{4} - \frac{k_{2}}{m_{1}}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)w^{2}\left(\frac{k_{1}}{m_{1}} - w^{2}\right) - \frac{c_{1}c_{2}^{2}w^{4}h_{0}}{m_{1}^{2}m_{2}^{2}} - \frac{k_{2}w^{2}}{m_{1}}\left(\frac{k_{1}}{m_{1}} - w^{2}\right)\left(\frac{k_{2}}{m_{2}} - w^{2}\right)\right]^{2}}\right]^{2}} \\ \theta = \arctan\frac{\frac{c_{1}c_{2}^{2}w^{5}h_{0}}{m_{1}m_{2}}\left(\frac{1}{m_{1}} + \frac{1}{m_{2}}\right)}{h_{0}w^{2}\left(\frac{k_{1}}{m_{1}} - w^{2}\right)\left(\frac{k_{2}}{m_{2}} - w^{2}\right)^{2} - \frac{k_{2}^{2}}{m_{1}^{2}}h_{0}w^{4} + \frac{k_{2}}{m_{1}}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)h_{0}w^{2}\left(\frac{k_{1}}{m_{1}} - w^{2}\right) - \frac{k_{2}}{m_{1}}h_{0}w^{4}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)}{m_{1}^{2}m_{2}^{2}} - \frac{k_{2}^{2}}{m_{1}^{2}}h_{0}w^{4} + \frac{k_{2}}{m_{1}}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)h_{0}w^{2}\left(\frac{k_{1}}{m_{1}} - w^{2}\right) - \frac{k_{2}}{m_{1}}h_{0}w^{4}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)}{m_{1}^{2}} + \frac{k_{2}^{2}}{m_{1}^{2}}h_{0}w^{4} + \frac{k_{2}}{m_{1}}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)h_{0}w^{2}\left(\frac{k_{1}}{m_{1}} - w^{2}\right) - \frac{k_{2}}{m_{1}}h_{0}w^{4}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)}{m_{1}^{2}} + \frac{k_{2}^{2}}{m_{1}^{2}}h_{0}w^{4} + \frac{k_{2}}{m_{1}}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)h_{0}w^{2}\left(\frac{k_{1}}{m_{1}} - w^{2}\right) - \frac{k_{2}}{m_{1}}h_{0}w^{4}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)}{m_{1}^{2}}} + \frac{k_{2}^{2}}{m_{1}^{2}}h_{0}w^{4} + \frac{k_{2}}{m_{1}}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)h_{0}w^{2}\left(\frac{k_{1}}{m_{1}} - w^{2}\right) - \frac{k_{2}}{m_{1}}h_{0}w^{4}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)}{m_{1}^{2}} + \frac{k_{2}^{2}}{m_{1}^{2}}h_{0}w^{4} + \frac{k_{2}}{m_{1}}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)h_{0}w^{2}\left(\frac{k_{1}}{m_{1}} - w^{2}\right) - \frac{k_{2}}{m_{1}}h_{0}w^{4}\left(\frac{k_{2}}{m_{2}} - w^{2}\right)}{m_{1}^{2}} + \frac{k_{2}}{m_{1}}h_{0}w^{4} + \frac{k_{2}}{m_{1}}h_{0}w^{4}\right)h_{0}w^{4} + \frac{k_{2}}{m_{1}}h_{0}$$

Introducing  $W = G \cos(\omega t) + H \sin(\omega t)$ , such that both sides of above equation are equal to W

$$\begin{cases} \frac{c_1}{m_1} \dot{z}_1 + \frac{k_1}{m_1} z_1 = W = G \cos(\omega t) + H \sin(\omega t) \\ \ddot{z}_2 + \left(\frac{c_2}{m_2} + \frac{c_2}{m_1}\right) \dot{z}_2 + \left(\frac{k_2}{m_1} + \frac{k_2}{m_2}\right) z_2 = W = G \cos(\omega t) + H \sin(\omega t) \end{cases}$$
(5)

The formula (6) can be solved by integration

$$z_{1} = \frac{m_{1}c_{1}}{k_{1}^{2} + m_{1}c_{1}\omega^{2}} \left[ \left( H\omega + \frac{k_{1}}{c_{1}}G \right) \sin(\omega t) + \left( \frac{k_{1}}{c_{1}}H - G\omega \right) \cos(\omega t) \right]$$
(6)

The second equation of formula (5) can be written as the summation of general solution of the corresponding homogeneous equation  $\overline{z}_2$ . A particular solution  $z_2^*$  of non-homogeneous equation

From above derivation, formula (13) is solved, formula (1) is introduced, and the expression of vehicle dynamic load of rough pavement can be described as

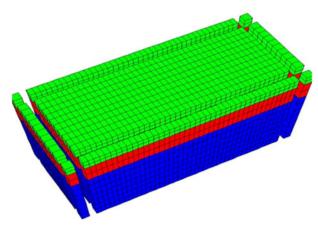
 $F(t) = P_0 + k_1 Z \sin(\omega t + \theta) + c_1 Z \omega \cos(\omega t + \theta)$ (12) where  $P_0 = (m_1 + m_2)g$ ; g = 9.8 N kg<sup>-1</sup>

#### Coupling of vehicle and pavement

Coupling effect of vehicle and pavement is reflected by the dynamic tire forces between vehicle and pavement, where tire force is not only related to the tire displacement and pavement roughness, but also is a function of the vertical vibration displacement of the pavement. It can be expressed as

$$F(t) = P_0 + k_1(y_1 - y_0 - w) + c_1(y_1 - y_0 - w)$$
(13)

where w is the vertical vibration displacement of pavement. As the vehicle does not pass through the pavement, it is considered that the vertical vibration displacement  $w_0=0$ , roughness waveform of pavement does not change. As the vehicle passing through the



1 Subgrade structure model

pavement, the roughness waveform of pavement changes to

$$y_1 = h_1 \sin(\omega t) \tag{14}$$

 $h_1$  can be written by  $h_1=h_0+w_0$ . Then, after the vehicle passes through the study pavement by *n* times, the roughness wavelength can be written as

$$y_{n} = (h_{0} + w_{0} + w_{1} + w_{2} + \dots + w_{n-1})\sin(\omega t)$$
 (15)

where  $w_1, w_2, w_3 \dots w_n$  is new roughness wavelengths of pavement based on the previous vehicle load, and they are applied on the subgrade structure.

### Numerical example

#### **Calculation model**

Figure 1 gives the subgrade model; the subgrade structure is divided into three layers. The surface layer and the base layer material is asphalt, and the subgrade material is clay. The vehicle single wheel load is simplified to be rectangular uniform load acting on the pavement. The length of the model is 9 m, and the width and depth is 3 and 8.6 m respectively. The boundary is set to be free. The thickness of the pavement is 0.2 m, while the thick of the base lawer and the subgrade is 0.4and 8 m repectviely. Material mechanics and physics calculation parameters<sup>11</sup> at different layers are shown in Table 1. The subgrade is constituted of homogeneous and isotropic elastic-plastic materials. Vehicle speed is  $30 \text{ m s}^{-1}$ , and the loads form is shown in equation (12). The pavement material is elastic-plastic, and initial roughness pavement amplitude of pavement is 0.02 m with the roughness wavelength of 3 m.

#### Vehicle loading

The vehicle single wheel load is simplified to be rectangular uniform load acting on the pavement. The length and the width of the rectangular load are 0.3 m. The load changes with time and is connected with the roughness waveform, where the change form is

Table 1 Calculation parameters of subgrade and pavement

Layer	Thickness/m	E/Mpa	μ	¢	c/kPa	ho/g cm <sup>-3</sup>
Surface layer	0.2	1400	0.25	60	350	2·5
Base layer	0.4	800	0.25	40	250	2.5
subgrade	8	60	0.35	22	55	1.9

described as equation (12). The load at different grid (i.e. in different pavement) is different. The calculate result at the previous step is applied to the calculation of the next step. In this study, the total step conduct cycled is set to be 30 in the model, and a finite difference method is used. The physical and mechanical characteristics of pavement structure layers are obtained from laboratory tests,<sup>12</sup> which are shown in Table 2.

In Table 2,  $m_2$  is the mass of back suspension of vehicle system,  $k_2$  is the spring stiffness of back suspension of vehicle system,  $c_2$  is the damping of back suspension,  $m_1$  is the mass of back unsuspension,  $k_1$ ,  $c_1$  is the stiffness and damping of tire respectively.

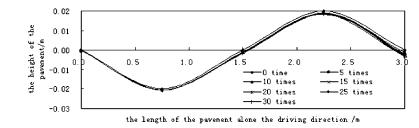
#### **Calculation analysis**

In this section, the coupling effect of vehicle and pavement is applied in the subgrade. Figure 2 shows the change of roughness waveform of pavement at different loading times. It can be seen that the deformation increased as the vehicle passed, but it is a sluggish increase course, the degree of the deformation is minimal. The deformation is more obvious in the peak of the wave than that in the trough region. In the first half wave period; the impact of vehicle load on the pavement is smaller than that when the vehicle is stationary. In the last half wave, the vehicle is in the state of overweight at the peak of the wave. Therefore, the deformation of pavement is larger. When the roughness of pavement is considered, the vertical deformation of the pavement varies. The situation of the pavement deformation must be judged with concerning of the roughness wavelength and the ampitude of pavement. Figure 3 shows the displacements of pavement in the peak and trough region under the vehicle load. At both peak and trough region, the variations of the dynamic deformation at different depths of the subgrade are consistent. The pavement deformation reaches the maximum value when the car is right in the place of the study pavement. The deformation of the subgrade decreases with the descending depth. After the vehicle passes, the deformations of the subgrade and pavement return to zero, and the pavement eventually return to the initial position, but it still has some residual deformation. The deformation of the pavement is large, when the vehicle travels in the pavement. The pavement deformation reached  $2 \cdot 4 \times 10^{-4}$  m, in the depth of  $1 \cdot 2$  m. The largest deformation of the subgrade is  $0 \cdot 4 \times 10^{-4}$  m, which is far smaller than that in the pavement.

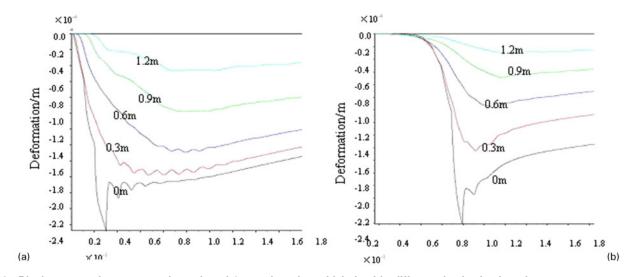
Figure 4 illustrates the comparison of the pavement roughness waveform with coupling and non-coupling calculation. The deformation tendency of the subgrade and pavement is the same for two cases, but it can be seen that the deformation of the pavement is larger in the coupling situation than that in the non-coupling

Table 2 Parameters of vehicle load

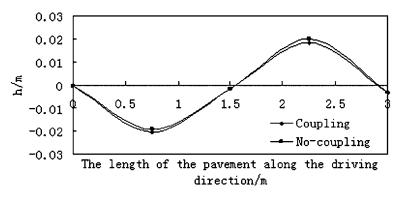
Parameter	No.	
m <sub>1</sub> /kg	473	
$k_1/\times 10^5 \text{ N m}^{-1}$	4.8	
$c_1 / \times 10^3 \text{ N s m}^{-1}$	8	
m <sub>2</sub> /kg	2885	
$k_2/\times 10^5$ N m <sup>-1</sup>	1.9	
$c_2/ \times 10^3 \text{ N s m}^{-1}$	14	



2 Changes of roughness waveform of pavement in different loading times



3 Displacement of pavement a in peak and b trough under vehicle load in different depth of subgrade



4 Comparison of pavement roughness waveform with coupling calculation and non-coupling calculation

situation at trough region. However, at the peak region, the deformation of the pavement is smaller in the coupling situation than that in the non-coupling situation. The vehicle load at this point is less than the static vehicle load, and the load in the coupling situation is less than that in the non-coupling situation. Oppositely, in the trough region, the vehicle is in the state of overweight, the load in the coupling situation is more than that in the non-coupling situation for trough case.

### Conclusion

In this paper, the study of subgrade dynamic response and deformation values under the action of multiple cyclic vehicle loads are carried out. The primary conclusions are drawn. 1. We provide a theoretical analysis and iterative calculation, where the expression of vehicle load considering the pavement roughness is introduced into the subgrade model.

2. Analysis of the subgrade dynamic response under the act of vehicle load in case of pavement roughness is also carried out. The deformations of the subgrade and the pavement under a number of vehicle loads are determined.

3. This study shows that, the deformation of the subgrade has a great relationship with amplitude and the wavelength of the pavement roughness. The vehicle loads influence the sunken position of the pavement. It is easy to produce a large deformation.

4. Coupling and non-coupling calculation results are compared in this study. The results show that the

deformations at different state of the pavement are different. At the peak region of the roughness waveform in the pavement, the deformation is smaller in the coupling calculation than that in non-coupling calculation, and vice versa at the trough region.

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### References

- S. M. Kim and B. F. McCullough: 'Dynamic response of plate on viscous Winkler foundation to moving loads of varying amplitude', *Eng. Struct.*, 2003, 25, 1179–1188.
- M. H. Huang and D. P. Thambiratnam: 'Deflection response of plate on winkler foundation to moving accelerated loads', *Eng. Struct.*, 2001, 23, 1134–1141.
- 3. D. Beskou and D. Dimitrios: 'Theodoralcopoulos. Dynamic effects of moving loads on road pavements: a review', *Soil Dynam. Earthquake Eng.*, 2011, (31), 547–567.

- 4. A. Loizos: 'Effects of user characteristics and vehicle type on road roughness perception', *Road Transp. Res.*, 1994, 4.
- Sijnl: 'Developing spectrum-based models for internat tonal roughness index and present serviceability index', J. Transport. Eng., 2001, 127, (6), 462–469.
- D. D. Theodorak opoulos: 'Dynamic analysis of a poroelastic halfplane soil medium under moving loads', *Soil Dynam. Earthquake Eng.*, 2003, 23, 521–533.
- S. M. Kim: 'Influence of horizontal resistance at plate bottom on vibration of plates on elastic foundation under moving loads', *Eng. Struct.*, 2004, 26, 519–529.
- H.-L. Yao, Z. Lu, X.-W. Luo and Y. Yang: 'Dynamic response of rough pavement on Kelvin foundation subjected to traffic loads', *Rock Soil Mech.*, 2009, **30**, (4), 890–896.
- H.-F. Zhou, J.-Q. Jiang and G.-H. Mao: 'Analysis of dynamic vehicle load caused by pavement roughness', *China Municip. Eng.*, 2002, (3), 10–13.
- Y.-Z. Guo, X.-L. Jin, D.-H. Ding and H.-J. Yang: 'Study and application of unstable traffic flow loads on rough pavement': *Rock. Soil Mech.*, 2007, 28, (2), 395–398.
- L. Tang, T. Xu Tong, P. Lin and H. Yu: 'Studies on dynamic stress characters of layered road system under traffic loading', *Chin. J. Rock Mech. Eng.*, 2009, 28, (Suppl. 2), 5.
- L. Z. Yao and H. L. Wu: 'Dynamic stress and deformation of a layered road structure under vehicle traffic loads: experimental measurements and numerical calculations', *Soil. Dynam. Earthquake Eng.*, 2012, 39, (3), 100–112.