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Probabilistic back analysis for geotechnical engineering based on Bayesian and support vector machine

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Abstract: Geomechanical parameters are complex and uncertain. In order to take this complexity and uncertainty into account, a probabilistic back-analysis method combining the Bayesian probability with the least squares support vector machine (LS-SVM) technique was proposed. The Bayesian probability was used to deal with the uncertainties in the geomechanical parameters, and an LS-SVM was utilized to establish the relationship between the displacement and the geomechanical parameters. The proposed approach was applied to the geomechanical parameter identification in a slope stability case study which was related to the permanent ship lock within the Three Gorges project in China. The results indicate that the proposed method presents the uncertainties in the geomechanical parameters reasonably well, and also improves the understanding that the monitored information is important in real projects.

Key words: geotechnical engineering; back analysis; uncertainty; Bayesian theory; least square method; support vector machine (SVM)

1 Introduction

Numerical models are used to study the geometry of the failure mechanisms and analyze the geotechnical engineering stability and design problems [1]. The accurate knowledge of the geomechanical parameters of the rock mass is a key factor in the numerical simulation. This is one of the most challenging tasks, and yet laboratory testing is an established method of determining the geomechanical parameters and it is limited by the relatively small scale of the laboratory test specimens [2]. In addition, the laboratory test results may be affected by sample disturbances [3]. Due to the complexity of the geomaterials, in situ testing techniques have been developed in order to overcome these limitations [4], but these techniques are often difficult to implement, costly and time-consuming.

The displacement of rock masses induced by an excavation can be measured relatively easily and reliably. The displacement-based back analysis is frequently used as a practical engineering tool to estimate the unknown geomechanical parameters. Many back-analysis approaches have been developed in the past 30 years [5–17]. However, the values obtained in this way are deterministic and do not interpret their uncertainty. In order to consider these uncertainties, the probabilistic back analyses have also been reported [18–20].

Probabilistic back analysis is a logical way of incorporating information from other sources, but it is more difficult to implement than traditional back-analysis methods [21]. In the present work, the Bayesian probability was combined with the displacement back analysis to provide a probabilistic back-analysis framework, which integrates the monitored displacement data and the uncertainty of the geomechanical parameters. The Bayesian approach has been applied to geotechnical engineering in past studies [21–26]. However, the Bayesian probability has rarely been incorporated into previous back analyses.

A detailed formulation of the LS-SVM algorithm is presented. Then, the probabilistic back analysis procedure incorporating the Bayesian probability and LS-SVM is presented. Finally, a case study is used to

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verify the proposed method.

2 Least squares support vector machine (LS-SVM)

The least squares support vector machine (LS-SVM) [27] is an alternative form of SVM regression. For a given training set of N data points $\{x_k, y_k\}$ ($k = 1, 2, \cdots, N$) with the input data $x_k \in \boldsymbol{R}^N$ and output $y_k \in \boldsymbol{r}$, where $\boldsymbol{R}^N$ is an N-dimensional vector space, and $\boldsymbol{r}$ is a one-dimensional vector space, the LS-SVM algorithm describes the model as

$$y(x)=\sum_{k=1}^{N}\alpha_k K(x,x_k)+b \tag{1}$$

where $K(x, x_k)$ is the kernel function, in which the general kernel function is the polynomial $K(X, Y)=((X\cdot Y)+1)^d$, d=1, 2, 3, $\cdots$; the radial kernel function is $K(X,Y)=\exp\left\{-\dfrac{|X-Y|^2}{\sigma^2}\right\}$; the radial function kernel (RBF) is $K(X, Y)=\tanh(\phi(X\cdot Y)+\theta)$; α_k are Lagrange multipliers; b is the scalar threshold. The values of α_k and b are obtained by

$$\begin{bmatrix} 0 & \boldsymbol{I}^{\mathrm{T}} \\ 1 & \boldsymbol{\Omega}+\gamma^{-1}\boldsymbol{I} \end{bmatrix}\begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{\alpha} \end{bmatrix}=\begin{bmatrix} 0 \\ \boldsymbol{y} \end{bmatrix} \tag{2}$$

where $\boldsymbol{y}=[y_1, \cdots, y_N]$; $\boldsymbol{I}=[1, \cdots, 1]$, $\boldsymbol{\alpha}=[\alpha_1, \cdots, \alpha_N]$; Mercer's theorem is applied within the $\boldsymbol{\Omega}$ matrix, $\boldsymbol{\Omega}=\boldsymbol{\varphi}(x_k)^{\mathrm{T}}\boldsymbol{\varphi}(x_l)=K(x_k, x_l)$, k, l=1, $\cdots$; and N. γ is the tolerance error**.** The analytical of $\boldsymbol{\alpha}$ and $\boldsymbol{b}$ is then given by

$$\begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{\alpha} \end{bmatrix}=\boldsymbol{\Phi}^{-1}\begin{bmatrix} 0 \\ \boldsymbol{y} \end{bmatrix} \tag{3}$$

where

$$\boldsymbol{\Phi}=\begin{bmatrix} 0 & \boldsymbol{I}^{\mathrm{T}} \\ 1 & \boldsymbol{\Omega}+\gamma^{-1}\boldsymbol{I} \end{bmatrix} \tag{4}$$

The algorithm was implemented with Excel Visual basic for Applications (VBA) software.

3 Probabilistic back analysis based on Bayesian probability and support vector machine

The numerical models and optimal methods are the key components of displacement back analysis. However, numerical modeling is time-consuming for large-scale projects. In this work, the LS-SVM represents the numerical model for mapping the relationship between the displacements and geomechanical parameters. A Microsoft Solver was used to search the geomechanical parameters as an optimizing method. The Bayesian method was used to present the probabilistic distribution law of the uncertainty of the geomechanical parameters. Figure 1 shows a flowchart of the proposed method.

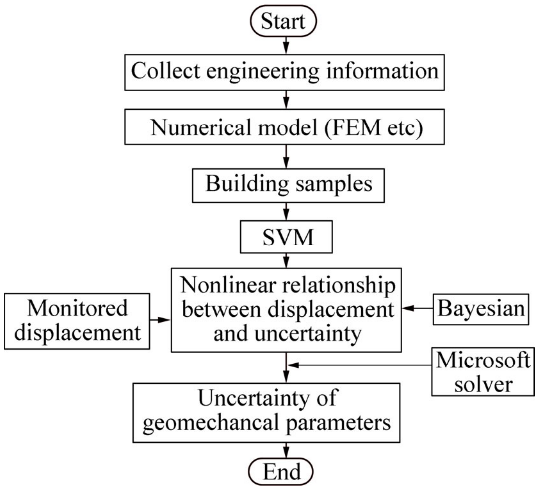

Fig. 1 Flowchart of probabilistic back analysis

3.1 LS-SVM based relationship between displacement and geomechanical parameters

The LS-SVM was used to map the nonlinear relationship between the geomechanical parameters, such as elastic modulus, cohesion, geostress coefficients, and monitored displacements. The mathematical model of the least squares support vector machine, $f_{\mathrm{LSSVM}}(\boldsymbol{X})$ is as follows:

$$f_{\mathrm{LSSVM}}(\boldsymbol{X}):\ \boldsymbol{R}^n\rightarrow\boldsymbol{R} \tag{5}$$

$$\boldsymbol{Y}=f_{\mathrm{LSSVM}}(\boldsymbol{X}) \tag{6}$$

$$\boldsymbol{X}=(x_1,\ x_2,\ \cdots,\ x_n) \tag{7}$$

$$\boldsymbol{Y}=(y_1,\ y_2,\ \cdots,\ y_n) \tag{8}$$

$$f_{\mathrm{LSSVM}}(\boldsymbol{X})=\sum_{k=1}^{N}\alpha_k K(\boldsymbol{X},X_k)+b_i \tag{9}$$

where x_i (i=1, 2, $\cdots$, n) represent the geomechanical parameters (deformation modulus, friction angle, geostress coefficients, etc); and y_i (i=1, 2, $\cdots$, n) are the displacements at the monitored sites.

A training process based on the known data set was needed in order to obtain $f_{\mathrm{LSSVM}}(\boldsymbol{X})$. The rock mass displacement at monitored sites, corresponding to the given set of the supposed geomechanical parameters, was calculated by numerical analysis (e.g., FEM model). The geomechanical parameters were adopted as the LS-SVM input, and the LS-SVM output was the displacement. The training process has been described by ZHAO [28].

3.2 Bayesian back analysis based on LS-SVM

In order to estimate the geomechanical parameters from the observed monitored displacements $Y_{\mathrm{mon}1}$, $Y_{\mathrm{mon}2}$, $\cdots$, $Y_{\mathrm{mon}k}$, the likelihood that the predicted displacements y_1, y_2, $\cdots$, y_k are equal to the corresponding measured displacement is a conditional probability density function (PDF) of θ [21]:

$$L(\theta \mid y_1 = Y_{\mathrm{mon}1}, y_2 = Y_{\mathrm{mon}1}, \cdots, y_k = Y_{\mathrm{mon}k}) = N_k[Y_{\mathrm{mon}1}/\delta_1(\theta), Y_{\mathrm{mon}2}/\delta_2(\theta), \cdots, Y_{\mathrm{mon}k}/\delta_k(\theta)] \tag{10}$$

where θ represents the estimated geomechanical parameter; $\delta_1(\theta)$, $\delta_2(\theta)$, $\cdots$, $\delta_k(\theta)$ are the predicted displacements at each monitored site using LS-SVM; N_k is the PDF of a multivariate normal distribution with a mean vector $[\boldsymbol{\mu}_\theta]=[\mu_1, \mu_2, \cdots, \mu_k,]$ and a covariance matrix as

$$[\boldsymbol{C}_\theta] = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \cdots & \sigma_{1k}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \cdots & \sigma_{2k}^2 \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{k1}^2 & \sigma_{k2}^2 & \cdots & \sigma_{kk}^2 \end{bmatrix} \tag{11}$$

where $\boldsymbol{C}_\theta$ is covariance matrix; σ_{k1k2}^2 is the covariance between θ_{k1} and θ_{k2}, $\sigma_{k1k2}^2 = \rho_\theta \mu_{k1}\mu_{k2}$, and ρ_θ is the correlation coefficient between the two model bias factors θ_{k1} and θ_{k2}. The posterior PDF of θ updated with the monitored displacement is obtained by [29]

$$f(\theta \mid y_1 = Y_{\mathrm{mon}1}, y_2 = Y_{\mathrm{mon}1}, \cdots, y_k = Y_{\mathrm{mon}k}) = m_k N_k[Y_{\mathrm{mon}1}/\delta_1(\theta), Y_{\mathrm{mon}2}/\delta_2(\theta), \cdots, Y_{\mathrm{mon}k}/\delta_k(\theta)] \cdot f(\theta) \tag{12}$$

where m_k is the normalization factor that guarantees a unity for the cumulative probability over the entire range of θ.

$$f(\theta) = \frac{\mathrm{e}^{(-\frac{1}{2}(\theta-\mu_\theta)\boldsymbol{C}_\theta^{-1}(\theta-\mu_\theta)^{\mathrm{T}})}}{(2\pi)^{\frac{k}{2}}\sqrt{\det(\boldsymbol{C}_\theta)}} \tag{13}$$

$$N_k(\boldsymbol{y}) = \frac{\mathrm{e}^{(-\frac{1}{2}(\frac{y}{\mu_d}-1)\boldsymbol{C}_\theta^{-1}(\frac{y}{\mu_d}-1)^{\mathrm{T}})}}{(2\pi)^{\frac{k}{2}}\sqrt{\det(\boldsymbol{C}_y)}} \tag{14}$$

$$[\boldsymbol{C}_y] = \begin{bmatrix} \mu_{y11}^2 & \mu_{y12}^2 & \cdots & \mu_{y1k}^2 \\ \mu_{y21}^2 & \mu_{y22}^2 & \cdots & \mu_{y2k}^2 \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{yk1}^2 & \mu_{yk2}^2 & \cdots & \mu_{ykk}^2 \end{bmatrix} \tag{15}$$

where k is the number of monitored sites; y is the predicted displacement; μ_θ are the mean values of the mean vector of the geotechnical parameters; and μ_d are the mean values of the mean vector for the displacements at the monitored sites; $\boldsymbol{C}_y$ is a covariance matrix of the monitored displacement at the different monitored sites; and $\mu_{yk1k2}^2 = \rho_y \mu_{yk1}\mu_{yk2}$.

The posterior distribution in the Bayesian statistics is obtained either by optimization or sampling techniques. In this work, optimization using Microsoft Excel Solver software was chosen. As the posterior mean was obtained, the posterior standard variation can also be calculated by

$$\sigma_\theta = \sqrt{\mathrm{diag}(\boldsymbol{C}_{\theta|d})} \tag{16}$$

where

$$\boldsymbol{C}_{\theta|d} = (\boldsymbol{G}^{\mathrm{T}}\boldsymbol{C}_y^{-1}\boldsymbol{G} + \boldsymbol{C}_\theta^{-1})^{-1} \tag{17}$$

and

$$[G] = \frac{\partial^2 f_{\mathrm{LSSVM}}(\theta)}{\partial\theta_j \partial\theta_k} \tag{18}$$

From Eq. (1) and Eq. (9), the first-order partial derivatives of the $f_{\mathrm{LSSVM}}(\theta)$ is calculated as [28]

$$\frac{\partial f_{\mathrm{LSSVM}}(\theta)}{\partial\theta_j} = \sum_{i=1}^{n} \alpha_i \frac{\partial K}{\partial\theta_j} \tag{19}$$

where θ_j is the jth variation of θ, and $\partial K/\partial\theta_j$ is the first partial derivative of the kernel function. When the kernel function is known, the first-order partial derivatives of the LS-SVM model can be readily calculated. If the kernel function is a radial kernel function, the first- order partial derivatives of the kernel function can be calculated as

$$\frac{\partial K}{\partial\theta_j} = -\frac{\theta_j - \theta_{ij}}{\sigma^2}\exp\left(-\frac{|\theta-\theta_i|^2}{2\sigma^2}\right) \tag{20}$$

where θ_j is the jth variation of θ, and θ_{ij} is the jth variation of θ_i.

The second-order partial derivatives of the LS-SVM model and kernel function can then be calculated from Eq. (19) and Eq. (20) as

$$\frac{\partial^2 f_{\mathrm{LSSVM}}(\theta)}{\partial\theta_j \partial\theta_k} = \sum_{i=1}^{n} \alpha_i \frac{\partial^2 K}{\partial\theta_j \partial\theta_k} \tag{21}$$

$$\frac{\partial^2 K}{\partial\theta_j \partial\theta_k} = \frac{(\theta_j - \theta_{ij})(\theta_k - \theta_{ik})}{\sigma^4}\exp\left(-\frac{|\theta-\theta_i|^2}{2\sigma^2}\right) \tag{22}$$

3.3 Computation procedure

The detailed procedure is as follows:

Step 1: The engineering information (geological conditions, dimensions, etc) was collected.

Step 2: From this information, the range of the parameters to be recognized was determined, and the input of training samples for the LS-SVM was built.

Step 3: The displacement of each sample was calculated from a numerical model.

Step 4: A sample set for the LS-SVM process was built. The tentative geomechanical parameters sets were estimated as follows. The displacement for each sample set, which had been previously found using a numerical model (e.g., finite element method), was determined for the LS-SVM model. The sample sets of the displacements at points on the numerical model corresponding to the measured displacements at the monitored sites, enabled the LS-SVM model in Eq. (1) to be obtained as the solution of Eq. (3).

Step 5: The Bayesian updating model was built (Eq. (12)).

Step 6: The posterior means of the recognized parameters were back-calculated using Excel Solver software.

Step 7: The posterior standard variation was computed using Eq. (16) to obtain the geomechanical parameters, as well as their uncertainty values.

4 Application examples

A permanent ship lock is one of the major components of the Three Gorges Project in China. It is one of the world's largest artificial navigational structures excavated in a rock mass. A single lock measuring 280 m×34 m×5 m deep was excavated in the granite. The vertical sidewall of the lock was 40 to 50 m high. The excavation was approximately 170 m at it deepest point. Both sides of the lock were high, with steep granite slopes. Section 17–17 is located at the head of the third lock (Fig. 2), and this is where the highest slope of the permanent ship lock area is located. The design and stability analysis of this slope were crucial for the construction of the lock. Obtaining the correct rock mass geomechanical parameters was a significant problem.

Some reports have suggested the use of regression equations [30] for calculating the stress fields from in situ measurements in the region. Another investigation on this project utilized a determistic back analysis, which is detailed in Ref. [14]. In the present work, a proposed probabilistic back analysis method was used. The parameters to be back-identified were the coefficients a_x and a_y for the geostress equation, and the deformation modulus for the four rock mass zones (moderately weathered rock; slightly weathered or fresh rock; unloading deformation; and damaged rock zones). The data for the deformation modulus, Poisson ratio, and weight for the strongly weathered zone, were provided by the Yangtze River Water Conservancy Committee. The rock masses in all the zones were considered to deform plastically. Their cohesion c and friction angle ϕ were determined directly from the engineering tests and previously monitored data [30].

In all, 50 sets of training samples were built using the FLAC established by ZHAO and YIN [14]. Each of the samples consisted of 6 LS-SVM inputs, which were the deformation modulus for the four rock mass zones defined above. The geostress coefficients a_x and a_y, and 6 LS-SVM outputs, which were displacements of six monitored sites, are listed in Table 1. Based on the LS-SVM algorithm described above, the LS-SVM code was written using Excel Visual Basic. The LS-SVM model was built, and its parameters α_i and b_i were obtained by using the training based on the training samples, as listed in Table 1. The first row is the b

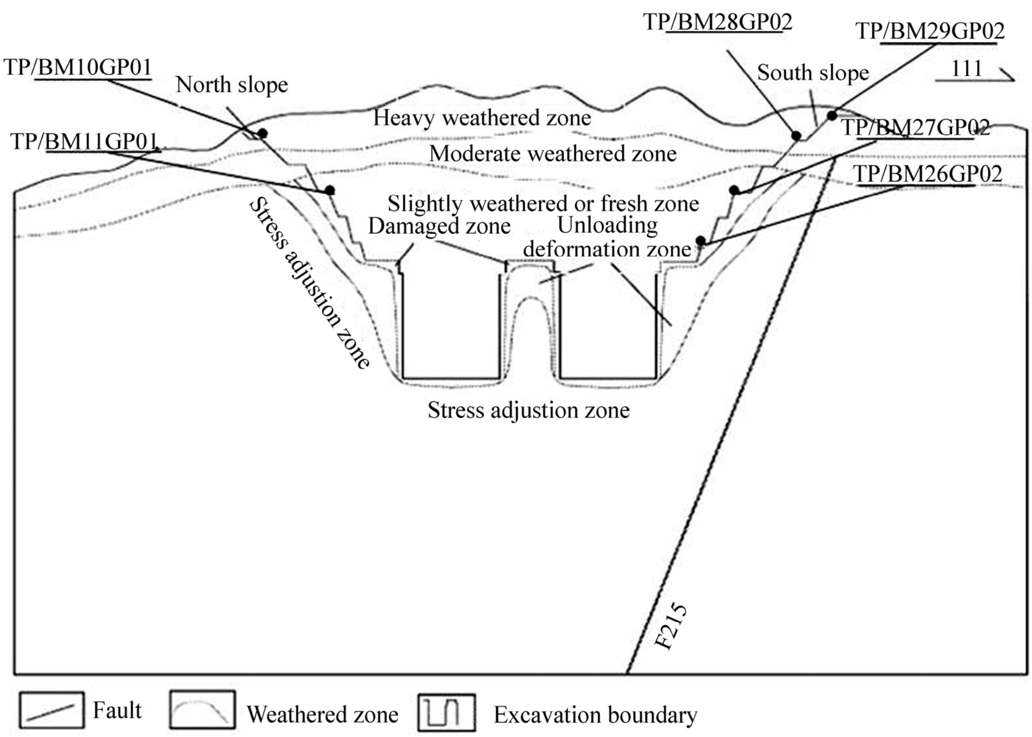

Fig. 2 Different rock zones and locations of monitored sites of Section 17–17

value of the LS-SVM model, and the other rows are the *a* value in the last six columns of Table 1. The values of the deformation modulus for the four rock mass zones defined above, and the geostress coefficients a_x and a_y were obtained by the LS-SVM model based on the observations at the six monitored sites, as listed in Table 2. The comparison of the calculated displacement and monitored displacement is illustrated. The six monitored sites are: TP/BM10GP01, TP/BM11GP01, TP/BM26GP02, TP/BM27GP02, TP/BM28GP02, and TP/BM29GP02, and their location is shown in Fig. 2.

The results obtained by the probabilistic and deterministic back analysis are compared in Table 2, in which the 2 monitored sites were TP/BM10GP01 and TP/BM11GP01; 4 monitored sites were TP/BM10GP01, TP/BM11GP01, TP/BM26GP02 and TP/BM27GP02; and six monitored sites were TP/BM10GP01, TP/BM11GP01, TP/BM26GP02, TP/BM27GP02, TP/BM28GP02, and TP/BM29GP02. A decrease in the standard deviation (s.d.) accompanying the increasing number of monitored sites was indicated. This illustrated the fact that the increased prior knowledge improves the understanding of the uncertainty of the geomechanical parameters. A comparison of the method based on SVM and the particle swarm optimization (PSO) [14] showed that the geomechanical parameters obtained by the deterministic methods were within the range of the proposed method, and suggested that the probabilistic back analysis provided considerable information regarding the geomechanical parameters, as well as providing a reasonable indication of the uncertainty of such parameters. The probabilistic back analysis considers that the uncertainty is more rational than the deterministic methods, and is consistent with the

Table 1 Training samples and LS-SVM model

No.	Training sample													Model of LSSVM					
	Updated parameter						Displacement of mointored sites/mm											
	Elastic modulus/GPa				CSS		TP/BM 10GP01	TP/BM 11GP01	TP/BM 26GP02	TP/BM 27GP02	TP/BM 28GP02	TP/BM 29GP02	α_{1i}	α_{2i}	α_{3i}	α_{4i}	α_{5i}	α_{6i}
	MWZ	DZ	UDZ	SWZ	a_x	a_y												
1	6.0	8.0	15.0	25.0	3.0	0.8	13.11	13.84	17.87	13.47	11.60	11.37	–153.14	–147.68	–188.75	–117.44	–124.02	–137.34
2	6.0	10.0	18.0	28.0	4.0	1.2	16.44	16.98	21.96	15.98	14.29	14.37	–48.08	–47.74	–60.92	–46.42	–39.70	–41.04
3	6.0	12.0	20.0	30.0	5.0	1.6	19.79	20.21	26.51	18.97	17.12	17.37	–12.97	–13.46	4.58	–25.03	–3.65	1.16
4	6.0	15.0	23.0	32.0	6.0	1.8	23.00	23.38	30.51	22.18	19.79	20.03	36.08	37.41	51.80	22.90	28.91	35.25
5	6.0	18.0	25.0	35.0	7.0	2.0	24.94	25.26	33.14	24.52	21.46	21.56	83.65	82.73	98.87	88.85	71.80	67.99
6	8.0	8.0	18.0	30.0	6.0	2.0	23.58	23.92	31.19	22.56	20.14	20.60	38.59	36.78	39.64	20.10	36.29	41.23
7	8.0	10.0	20.0	32.0	7.0	0.8	26.28	26.62	35.16	25.94	22.43	22.73	67.21	71.79	102.21	97.89	60.28	52.30
8	8.0	12.0	23.0	35.0	3.0	1.2	9.56	10.02	13.18	9.63	8.23	8.30	–65.09	–63.12	–94.12	–53.81	–56.19	–62.10
9	8.0	15.0	25.0	25.0	4.0	1.6	18.70	19.17	23.96	17.79	16.07	16.26	–68.18	–69.50	–113.39	–77.77	–68.26	–60.46
10	8.0	18.0	15.0	28.0	5.0	1.8	20.92	21.40	28.45	19.93	17.89	18.23	–55.69	–55.41	–62.97	–65.66	–43.04	–37.97
11	10.0	8.0	20.0	35.0	4.0	1.8	12.81	13.17	17.74	12.33	10.84	11.23	–42.43	–44.71	–56.94	–59.81	–35.92	–33.09
12	10.0	10.0	23.0	25.0	5.0	2.0	23.15	23.45	30.17	21.79	19.89	20.22	–90.26	–93.48	–103.68	–102.43	–74.03	–71.20
13	10.0	12.0	25.0	28.0	6.0	0.8	25.47	25.78	33.86	24.73	21.94	22.13	42.32	42.53	89.95	51.01	45.24	40.19
14	10.0	15.0	15.0	30.0	7.0	1.2	27.62	28.04	37.67	26.98	23.32	23.75	108.71	110.82	156.24	119.65	84.96	86.05
15	10.0	18.0	18.0	32.0	3.0	1.6	10.38	10.89	14.24	10.15	8.73	8.94	–122.54	–121.17	–188.27	–117.64	–113.71	–114.08
16	12.0	8.0	23.0	28.0	7.0	1.6	29.31	29.51	38.13	28.24	24.94	25.44	98.40	95.17	101.64	99.71	78.69	82.23
17	12.0	10.0	25.0	30.0	3.0	1.8	10.87	11.37	14.66	10.59	9.17	9.40	–108.66	–107.25	–161.51	–102.82	–101.21	–103.38
18	12.0	12.0	25.0	32.0	4.0	2.0	13.80	14.26	19.34	13.39	11.66	12.02	–6.27	–2.98	16.39	–14.19	–15.45	–9.98
19	12.0	15.0	28.0	35.0	5.0	0.8	16.31	16.73	23.62	16.51	14.11	14.33	–25.03	–24.22	5.71	–15.83	–21.07	–19.66
20	12.0	18.0	20.0	25.0	6.0	1.2	28.17	28.47	37.48	26.95	24.17	24.39	134.23	126.80	182.71	120.99	120.20	119.24
21	15.0	8.0	25.0	32.0	5.0	1.2	17.49	17.88	24.09	17.17	15.11	15.48	9.17	8.17	13.34	4.94	17.34	17.12
22	15.0	10.0	15.0	35.0	6.0	1.6	19.39	19.82	27.55	19.17	16.42	16.95	4.26	4.00	5.01	0.38	0.74	4.15
23	15.0	12.0	18.0	25.0	7.0	1.8	32.12	32.48	42.73	30.70	27.36	27.77	187.62	186.52	241.28	174.27	158.59	158.99

To be continued

Continued

No.	Training sample												Model of LS-SVM					
	Updated parameter						Displacement of mointored sites/mm											
	Elastic modulus/GPa				CSS		TP/BM 10GP01	TP/BM 11GP01	TP/BM 26GP02	TP/BM 27GP02	TP/BM 28GP02	TP/BM 29GP02	α_{1i}	α_{2i}	α_{3i}	α_{4i}	α_{5i}	α_{6i}
	MWZ	DZ	UDZ	SWZ	a_x	a_y												
24	15.0	15.0	20.0	28.0	3.0	2.0	11.45	12.05	15.58	11.22	9.66	9.85	–125.61	–123.66	–181.48	–119.65	–114.40	–115.96
25	15.0	18.0	23.0	30.0	4.0	0.8	14.85	15.40	20.58	14.98	13.00	13.00	–7.16	–6.03	–13.04	3.84	7.26	–2.18
26	6.0	8.0	15.0	32.0	7.0	1.8	26.28	26.63	34.85	25.67	22.25	22.69	145.84	144.12	177.20	146.19	118.29	121.62
27	6.0	10.0	18.0	35.0	3.0	2.0	9.56	9.98	13.05	9.19	7.97	8.20	–103.70	–105.69	–143.95	–111.88	–95.74	–94.94
28	6.0	12.0	20.0	25.0	4.0	0.8	18.65	19.29	24.48	18.36	16.39	16.22	–44.10	–37.01	–60.71	–15.87	–29.30	–42.46
29	6.0	15.0	23.0	28.0	5.0	1.2	21.51	21.99	28.41	20.76	18.75	18.80	–2.82	–0.72	12.05	–1.62	5.58	3.22
30	6.0	18.0	25.0	30.0	6.0	1.6	24.75	25.16	32.44	23.81	21.39	21.50	80.09	79.92	90.99	62.65	67.16	69.37
31	8.0	8.0	18.0	25.0	5.0	1.6	23.00	23.43	30.34	21.76	19.86	20.20	98.17	94.60	129.01	67.52	83.90	95.50
32	8.0	10.0	20.0	28.0	6.0	1.8	25.50	25.79	33.48	24.26	21.80	22.19	47.23	43.63	55.86	35.54	35.02	38.44
33	8.0	12.0	23.0	30.0	7.0	2.0	28.35	28.54	36.92	27.24	24.13	24.51	85.99	80.83	74.98	80.47	62.76	64.54
34	8.0	15.0	25.0	32.0	3.0	0.8	10.58	11.14	14.62	11.21	9.46	9.21	–144.60	–141.39	–184.13	–100.57	–111.18	–128.05
35	8.0	18.0	15.0	35.0	4.0	1.2	13.13	13.62	18.59	13.01	11.23	11.41	–53.74	–53.21	–61.89	–50.27	–40.19	–43.09
36	10.0	8.0	20.0	30.0	3.0	1.2	10.77	11.32	14.79	10.65	9.20	9.37	–128.61	–124.18	–159.45	–110.31	–112.04	–117.39
37	10.0	10.0	23.0	32.0	4.0	1.6	14.17	14.55	19.12	13.62	12.06	12.41	0.46	–1.60	–4.36	–14.25	0.94	6.38
38	10.0	12.0	25.0	35.0	5.0	1.8	16.69	16.98	22.79	16.05	14.29	14.72	29.97	25.32	22.60	–0.26	27.27	36.98
39	10.0	15.0	15.0	25.0	6.0	2.0	28.10	28.49	37.46	26.54	23.92	24.30	109.60	106.67	138.89	85.81	90.88	94.94
40	10.0	18.0	18.0	28.0	7.0	0.8	29.90	30.24	40.13	29.02	25.44	25.70	111.83	112.23	148.40	124.80	89.60	85.45
41	12.0	8.0	23.0	35.0	6.0	0.8	19.95	20.27	27.42	19.93	17.05	17.42	42.46	43.43	61.73	63.82	37.26	33.18
42	12.0	10.0	25.0	25.0	7.0	1.2	32.97	33.19	42.63	31.60	28.19	28.55	248.28	245.48	298.42	239.25	210.07	209.96
43	12.0	12.0	15.0	28.0	3.0	1.6	11.41	12.01	15.71	11.13	9.60	9.82	–176.51	–176.67	–256.23	–172.60	–159.53	–159.93
44	12.0	15.0	18.0	30.0	4.0	1.8	14.92	15.39	20.46	14.44	12.68	13.00	–40.14	–40.21	–56.39	–40.31	–28.73	–27.60
45	12.0	18.0	20.0	32.0	5.0	2.0	18.10	18.42	24.97	17.36	15.41	15.82	23.22	17.65	44.55	–0.04	19.84	28.92
46	15.0	8.0	25.0	28.0	4.0	2.0	15.59	16.07	21.00	15.07	13.34	13.66	–119.09	–116.55	–146.49	–116.88	–101.34	–101.93
47	15.0	10.0	15.0	30.0	5.0	0.8	18.26	18.87	26.34	18.28	15.79	16.08	–33.26	–27.93	–1.42	–15.95	–14.68	–18.64
48	15.0	12.0	18.0	32.0	6.0	1.2	21.38	21.77	30.00	21.02	18.23	18.65	26.68	24.74	45.49	22.83	25.39	24.54
49	15.0	15.0	20.0	35.0	7.0	1.6	23.37	23.66	32.48	23.08	19.76	20.26	73.75	72.58	94.27	77.41	57.05	58.99
50	15.0	18.0	23.0	25.0	3.0	1.8	12.86	13.59	17.29	12.74	11.00	11.16	–156.11	–148.34	–203.71	–141.52	–137.94	–135.44

Note: b_1=19.32, b_2=19.85, b_3=26.28, b_4=19.18, b_5=16.67, b_6=16.80; MWZ—Moderately weathered zone; DZ—Damaged zone; UDZ—Unloading deformation zone; SWZ—Slightly weathered or fresh zone; CSS—Coefficient of in-situ stress.

Table 2 Comparison of probabilistic and deterministic back analyses

Method	Monitored sites number	Statistic parameter	Deformation Modulus/GPa				Coefficient of in-situ stress	
			MWZ	DZ	UDZ	SWZ	a_x	a_y
Probabilistic back analysis	2	Mean value	10.039	11.988	19.801	31.218	4.782	1.599
		Standard deviation	2.990	3.582	5.553	2.290	0.486	0.479
	4	Mean value	10.010	11.900	20.042	31.447	4.692	1.602
		Standard deviation	2.252	3.444	3.047	0.874	0.133	0.161
	6	Mean value	9.030	12.018	20.755	31.062	4.732	1.602
		Standard deviation	0.424	0.933	0.665	0.300	0.046	0.106
Deterministic back analysis [14]	6	Mean value	6.000	9.498	17.313	29.253	4.355	1.370
		Standard deviation	—	—	—	—	—	—

Note: MWZ—Moderately weathered zone; DZ—Damaged zone; UDZ—Unloading deformation zone; SWZ—Slightly weathered or fresh zone.

complexity and uncertainty of the geotechnical engineering difficulties.

For this particular case study, the probability density distribution of the geomechanical parameters based on the non-observations and observations from the 2, 4, and 6 monitored sites, are shown in Fig. 3. In Fig. 3, it can be seen that the certainty of the geomechanical parameters increases with the increasing of the monitored information, and the geomechanical parameters are close to the real value. The displacement calculated by the LS-SVM, based on the geomechanical parameters from the 2, 4, and 6 monitored sites, as well as the monitored displacement are shown in Fig. 4. It is clear from Fig. 4 that the proposed back analysis method can produce high accuracy results in this case study. Such results can be used to guide the design of the geotechnical engineering structures. The distributions of the calculated displacement at the two monitored sites are shown in Fig. 5. The uncertainty of the displacement values is highlighted, and this indicates that a greater amount of monitored information (prior knowledge) potentially reduces the effect of this uncertainty.

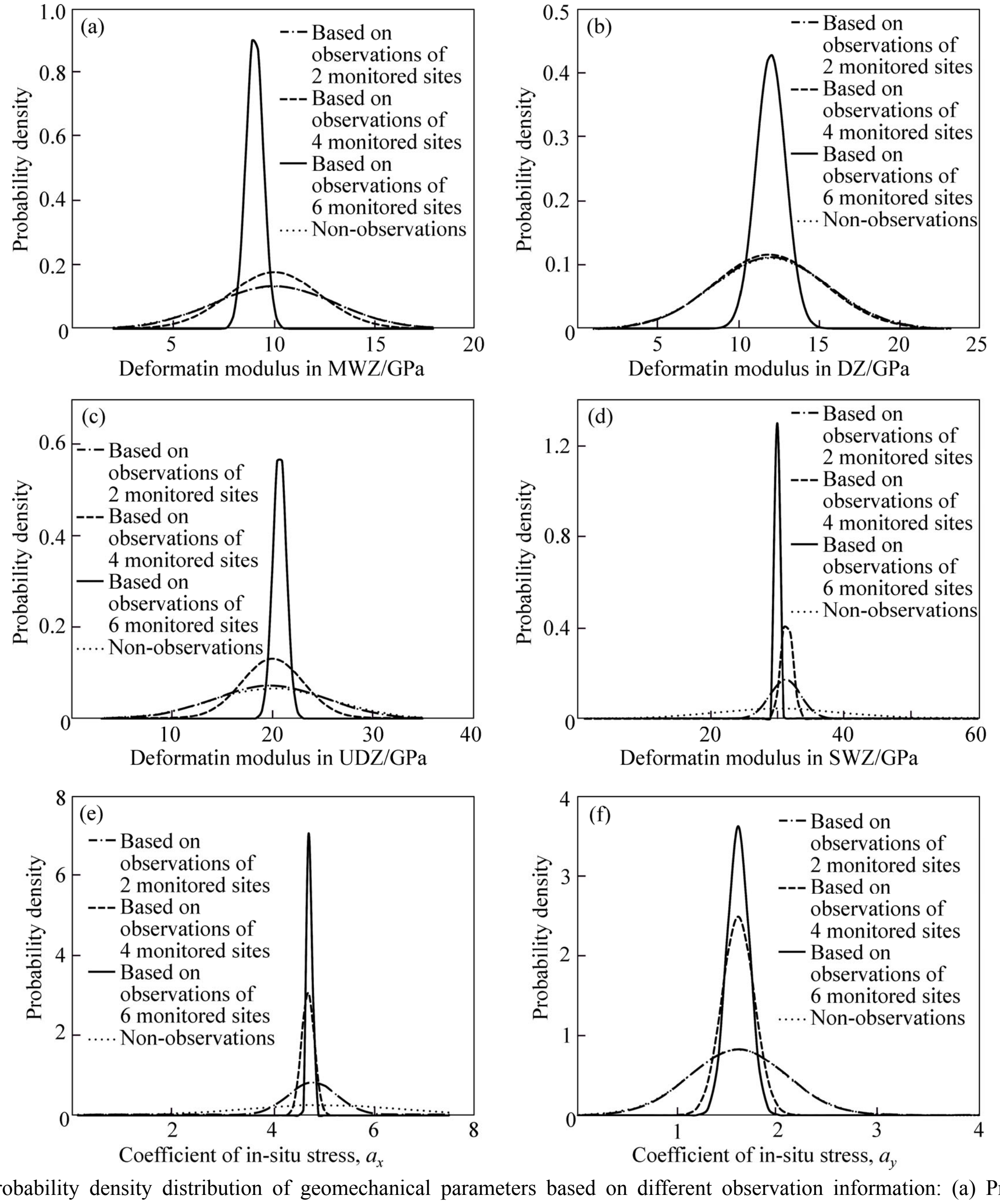

Fig. 3 Probability density distribution of geomechanical parameters based on different observation information: (a) Probability density distribution of deformation modulus in MWZ; (b) Probability density distribution of deformation modulus in DZ; (c) Probability density distribution of deformation modulus in UDZ; (d) Probability density distribution of deformation modulus in SWZ; (e) Probability density distribution of coefficient of in-situ stress a_x; (f) Probability density distribution of coefficient of in-situ stress a_y

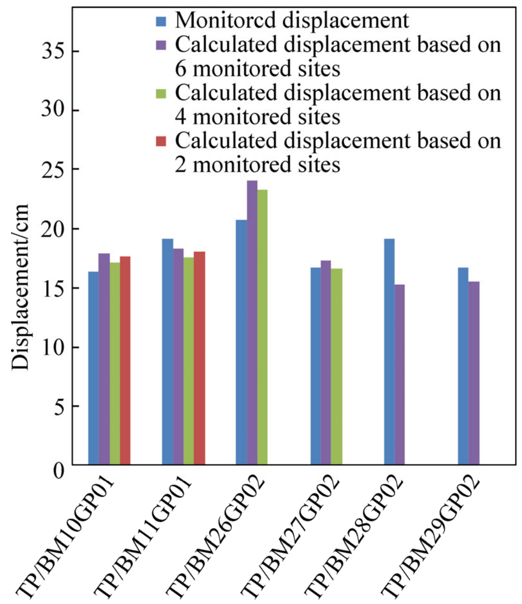

Fig. 4 Comparison of calculated displacement by LS-SVM, and monitored displacement based on different observation information

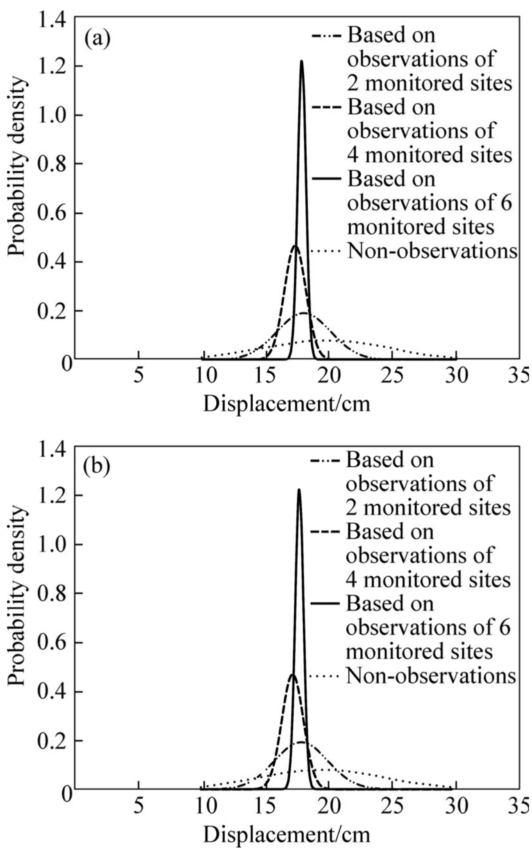

Fig. 5 Displacement distributions of monitored sites TP/BM11GP01 and TP/BM11GP01 based on different observation information: (a) TP/BM10GP01; (b) TP/BM11GP01

5 Conclusions

This work describes a proposed probabilistic back analysis that combines the Bayesian probability with a LS-SVM in order to illustrate the uncertainties in the rock mass mechanical properties used for geotechnical engineering purposes. A LS-SVM model implemented in Microsoft Excel VBA is used in back-analysis of the relationship between the geomechanical parameters and displacement in the rock masses. Microsoft Solver software is utilized to interrogate the geomechanical parameters. The method is found to be easy to use and was readily understood by the field engineers. The proposed approach is applied to the geomechanical parameter identification in a slope stability case study, which is related to the permanent ship lock within the Three Gorges project in China. These results are compared with the deterministic results obtained earlier.

The probabilistic back analysis produces more information than the deterministic back analysis, and characterizes the behavior of the rock masses in the geotechnical engineering problems reasonably well. The distributions of the geomechanical parameters are compared with the observed values of the deformation measured at the different numbers within the monitored sites. The uncertainties of the values of the geomechanical parameters are reduced when more measurement information is added. Therefore, the probabilistic back analysis can be used in reliability-based designs (RBD). The proposed method is useful and assists in the stability analysis, slope design, and construction safety.

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