Engineering Analysis with Boundary Elements 73 (2016) 61–68

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A new dual reciprocity hybrid boundary node method based on Shepard and Taylor interpolation method and Chebyshev polynomials

Fei Yan*, Xia-Ting Feng, Jia-He Lv, Peng-Zhi Pan, Shao-Jun Li

State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan 430071, China

ARTICLE INFO

Article history:
Received 28 June 2016
Received in revised form
9 August 2016
Accepted 31 August 2016

Keywords:
Meshless method
Shepard and Taylor interpolation method
Dual reciprocity hybrid boundary node method
Dual reciprocity method
Chebyshev polynomials

ABSTRACT

A new dual reciprocity hybrid boundary node method (DHBNM) is proposed in this paper, in which the Shepard and Taylor interpolation method (STIM) and Chebyshev polynomials interpolation are proposed. Firstly, the Shepard interpolation is used to construct zero level shape function, and the high-power shape functions are constructed through the Taylor expansion, and through those two methods, no inversion is needed in the whole process of the shape function construction. Besides, Chebyshev polynomials are used as the basis functions for particular solution interpolation instead of the conical function, radial basis functions, and the analytical solutions of the basic form of particular solutions related to Chebyshev polynomials for elasticity are obtained, by means of this method, no internal node is needed, and interpolation coefficients can be given as explicit functions, so no inversion is needed for particular solution interpolation, which costs a large amount of computational expense for the traditional method. Based on those two methods, a new dual reciprocity hybrid boundary node method is developed, compared to the traditional DHBNM, no inversion is needed for both shape function construction and particular solution interpolation, which greatly improves the computational efficiency, and no internal node is needed for particular solution interpolation. Numerical examples are given to illustrate that the present method is accurate and effective.

1. Introduction

In the past 30 years, meshless methods have been developed rapidly, and a lot of different meshless methods have been proposed one after another, and some of them have been widely used in practical engineering. At the same time, for constructing different meshless methods, a lot of shape function constructing methods for different meshless methods have been proposed, such as: the moving least square(MLS), the point interpolation method, the Kriging interpolation and so on [15]. As a widely used approximation method, MLS has been firstly proposed by Lancaster and Salkauskas [14], and then has been widely applied for different kinds of meshless methods, such as the element-free Galerkin method (EFG) [3], the meshless local Petrov-Galerkin method (MLPG) [2,11], the local boundary integral equation method (LBIM) [1,43], the boundary node method (BNM) [24] and so on. In those methods, although no element is needed for the variable interpolation, background elements are inevitable for 'energy' integration. Besides, some other methods such as mesh regeneration algorithm [12,39] and meshless singular boundary method have been developed recently.

In order to overcome the defect of background element for the boundary node method, applying hybrid displacement variational formulation and three fields interpolation scheme, Zhang and Yao [40,41] proposed the hybrid boundary node method(HBNM) and the regular hybrid boundary node method(RHBNM). Later, based on HBNM and rigid body displacement, Miao and Wang [19,20] developed a meshless method of singular hybrid boundary node method(SHBNM), later, they applied this method for analysis of reinforced concrete members and 3D composite materials [21–23]; and furthermore, applying dual reciprocity method (DRM) [32] into SHBNM, [36,37,38] proposed the dual reciprocity hybrid boundary node method(DHBNM) to solve inhomogeneous, dynamic, nonlinear problems, and so on.

The HBNM, RHBNM, SHBNM and DHBNM are MLS-based meshless methods. As an approximation method, the shape function based on MLS lacks the Delta function property compared with the widely used shape function obtained by interpolation, so boundary conditions cannot be imposed easily and directly, and its frequently inversion operation is inefficient. Aimed to those defects, the radial basis function interpolation method [16, 42], Kriging interpolation method [30], partition of unity [18] have been widely used to construct meshless shape function in past decades,

* Corresponding author.

http://dx.doi.org/10.1016/j.enganabound.2016.08.015
0955-7997/

and Cai and Zhu [5,6] have proposed meshless Shepard interpolation method, which satisfies the Delta function property and has high order completeness. Based on the Shepard interpolation method and Taylor expansion, [33] proposed the Shepard and Taylor interpolation method(STIM). As a shape function constructing method, the advantages of STIM are: the interpolation property, the arbitrarily high order consistency, no inversion for the whole process of shape function constructing, and the low computational expense.

To avoid domain integral that comes out from the inhomogeneous term of governing equation, the dual reciprocity method(DRM) was introduced by Nardini and Brebbia [26,27] in 1982 for elastodynamic problems and extended by Wrobel and Brebbia [32] to time-dependent diffusion in 1986. Later, a book by Partridge et al. [28] has been published to introduce dual reciprocity method to apply for boundary element method. In the first few years, the conical function $1+r$ was exclusively employed for approximation of inhomogeneous term. After that the theory of radial basis function for DRM was introduced by Golberg and Chen [9] to replace the conical function. Since then, a lot of important papers about DRM have been focused on the investigation of the effect of choosing different radial basis functions [7,25].

Actually, a good choice of radial basis functions improves the accuracy and efficiency of DRM, and it is widely accepted as a reliable numerical method in transferring the domain integral to the boundary in the BEM community. But in those methods, the inversion for DRM processes is inevitable, which costs much large computational time, and it is inefficient for large scale calculation. It is well-known that Chebyshev polynomials are valuable tools in numerical analysis and approximation theory [17], and they are widely used in the numerical solution of boundary value problems for partial differential equations with spectral methods [4], which has a rapid convergence rate. Golberg et al. [10] used the symbol software mathematica to connect monomials with Chebyshev polynomials and employed their derived particular solution for floating number computing. Then Reutskiy and Chen [29] circumvented the tedious book keeping by using two-stage approximations of trigonometric functions and Chebyshev polynomials. Later, [13] used Chebyshev polynomials for approximating particular solutions of elliptic equations, and Tsai [31] took his effort for the particular solutions of Chebyshev polynomials for Reissner plates under arbitrary loadings.

In this paper, in order to overcome the inefficient property of the traditional dual reciprocity hybrid boundary node method (DHBNM), a new dual reciprocity hybrid boundary node method (DHBNM) is proposed, in which the Shepard and Taylor interpolation method(STIM) is employed for shape function constructing, and Chebyshev polynomials are applied for basis functions of particular solution interpolation. Firstly, the Shepard interpolation is used to construct zero level shape function, and the high-power shape functions are constructed through the Taylor expansion, and through those methods, STIM is developed, and no inversion is needed in the process of the shape function construction, and much lower computational expense is achieved. At the same time, Chebyshev polynomials are used as basis functions for particular solution interpolation instead of the conical function, radial basis functions, by means of this method no real internal interpolation node is needed, and the interpolation coefficients can be given as explicit functions, then no inversion is needed in the process of particular solution interpolation, which costs a large amount of computational expense for the traditional method. Based on those two methods and hybrid boundary node method, a new dual reciprocity hybrid boundary node method is developed, compared to the traditional DHBNM, no inversion is needed for both shape function construction and particular solution interpolation process, which greatly improve the computational efficiency.

2. Description of governing equation

In this paper, we take elasticity problem as the example, then consider an elasticity problem in domain Ω bounded by Γ. The governing equation can be given

$$\sigma_{ij,j} = b_i \tag{1}$$

$$u_i = \hat{u}_i \text{ on } \Gamma_u \tag{2}$$

$$t_i = \sigma_{ij} n_j = \hat{t}_i \text{ on } \Gamma_t \tag{3}$$

In which the superposed bar denotes the prescribed boundary values and $\mathbf{n}$ is the unit vector of outward normal of boundary, b_i is the inhomogeneous term.

According to the traditional DHBNM theory, the solution variable of displacement u can be divided into the complementary solution u^c and the particular solution u^p, which can be expressed as [36–38]

$$u_i = u_i^c + u_i^p \tag{4}$$

The complementary solution u_i^c must satisfy the homogeneous equation and the modified boundary conditions, but the particular solution just satisfies the inhomogeneous equation in the whole space.

The particular solution u_i^p can be solved by Chebyshev polynomials interpolation in Section 4 by means of dual reciprocity method. The complementary solution u^c must satisfy the homogeneous equation and the modified boundary conditions, according to modified variational principle of hybrid boundary node method, we can get the local integral of the present method,

$$\int_{\Gamma_s} (t-\tilde{t}) h_J(Q) d\Gamma - \int_{\Omega_s} \sigma_{ij,j} h_J(Q) d\Omega = 0 \tag{5}$$

$$\int_{\Gamma_s} (u-\tilde{u}) h_J(Q) d\Gamma = 0 \tag{6}$$

In which Γ_s is the interaction of sub-domain Ω_s and the boundary of the calculation domain, which can be seen in references [36–38], and test function $h_J(Q)$ in the present method is given as

$$h_J(Q) = \begin{cases} \dfrac{\exp\left[-(d_J/c_J)^2\right] - \exp\left[-(r_J/c_J)^2\right]}{1-\exp\left[-(r_J/c_J)^2\right]} & 0 \le d_J \le r_J \\ 0 & d_J \ge r_J \end{cases} \tag{7}$$

in which the variables, the factors and its related contents can be referred in [36–38].

3. Shepard and Taylor interpolation method

MLS is a widely used shape function construction method for meshless methods, and as an approximation method, MLS has high accuracy, but it has three disadvantages, firstly, it is lack of the Delta function property, so the boundary condition cannot be easily and directly imposed; secondly, high computational expense is needed, because individual interpolation coefficients are needed for every interpolation nodes; finally, the inversion is inevitable for every nodes in their shape function constructing processes. To overcome those defects, the Shepard and Taylor interpolation method is developed in the present method.

According to the Shepard function interpolation theory, the Shepard function interpolation can be written as [33,5,6]

$$u(x, y) = \sum_{i=1}^{M} \varphi_i^{\,0}(x, y)u_i \tag{8}$$

in which $\varphi_i^{\,0}(x, y)$ can be given as

$$\varphi_i^{\,0}(x, y) = \frac{w_i(x, y)}{\sum_{j=1}^{N} w_j(x, y)} \tag{9}$$

In order to ensure the weight function satisfy the Kronecker delta function property, the weight function can be chosen as [33,5,6]

$$w_i(x, y) = \begin{cases} \dfrac{d_i^2}{r_i^2 + \varepsilon}\cos^2\left(\dfrac{\pi r_i}{2d_i}\right) & 0 \le r_i \le d_i \\ 0 & r_i > d_i \end{cases} \tag{10}$$

In which the r_i, d_i, ε can be referred as [33]. It is shown that this shape function satisfies zero order consistency, and can satisfy the Kronecker delta function property if the weight function $w_i(x, y)$ is singular at $(x, y) = (x_i, y_i)$. But we can also see that this shape function has no high order consistency, we will use the Taylor expansion to construct the high order consistency of the present shape function.

According to the Taylor expansion, the variable $u(x, y)$ can be expanded at Gauss integral point (x_0, y_0) via the Taylor expansion [33]

$$\begin{aligned} u(x, y) = {} & u(x_0, y_0) + \left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)u(x_0, y_0) \\ & + \frac{1}{2!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 u(x_0, y_0) \\ & + \ldots + \frac{1}{n!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^n u(x_0, y_0) \\ & + \frac{1}{(n+1)!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^{n+1} u(x_0 + \beta h, y_0 + \beta k) \end{aligned} \tag{11}$$

in which $0 < \beta < 1$, $h = x - x_0$, $k = y - y_0$, and (x_0, y_0) is the Gauss point of each integral sub-domain.

Employed the Shepard function interpolation, the field function $u(x_0, y_0)$ at integral node (x_0, y_0) can be rewritten as

$$u(x_0, y_0) = \sum_{i=0}^{M} \varphi_i^{\,0}(x_0, y_0)u_i(x_i, y_i) \tag{12}$$

$$\frac{\partial^m u(x_0,y_0)}{\partial x^{m-r}\partial y^r} = \sum_{i=0}^{M} \frac{\partial^m \varphi_i^{\,0}(x_0,y_0)}{\partial x^{m-r}\partial y^r}u_i(x_i,y_i) = \sum_{i=0}^{M} \varphi^0_{i,x^{m-r}y^r}(x_0,y_0)u_i(x_i,y_i) \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (11), then Eq. (11) can be rewritten as

$$\begin{aligned} u(x, y) = \sum_{i=0}^{M}\Bigg[& \varphi_i^{\,0} + \varphi_{i,x}^0 \sum_{j=0}^{M} \varphi_j^0(x - x_j) + \varphi_{i,y}^0 \sum_{j=0}^{M} \varphi_j^0(y - y_j) \\ & + \frac{1}{2}\varphi_{i,xx}^0 \sum_{j=0}^{M} \varphi_j^0(x - x_j)^2 \\ & + \frac{1}{2}\varphi_{i,yy}^0 \sum_{j=0}^{M} \varphi_j^0(y - y_j)^2 + \varphi_{i,xy}^0 \sum_{j=0}^{M} \varphi_j^0(x - x_j)(y - y_j) \\ & + \ldots + \frac{1}{2}\varphi_{i,xyy}^0 \sum_{j=0}^{M} \varphi_j^0(x - x_j)^1(y - y_j)^2 + \ldots \Bigg]u_i(x_i, y_i) \end{aligned} \tag{14}$$

From Eq. (14), the shape function of the present method can be written as [33]

$$\begin{aligned} \Phi_i(x, y) = {} & \varphi_i^{\,0} + \varphi_{i,x}^0 \sum_{j=0}^{M} \varphi_j^0(x - x_j) + \varphi_{i,y}^0 \sum_{j=0}^{M} \varphi_j^0(y - y_j) \\ & + \frac{1}{2}\varphi_{i,xx}^0 \sum_{j=0}^{M} \varphi_j^0(x - x_j)^2 \\ & + \frac{1}{2}\varphi_{i,yy}^0 \sum_{j=0}^{M} \varphi_j^0(y - y_j)^2 \\ & + \varphi_{i,xy}^0 \sum_{j=0}^{M} \varphi_j^0(x - x_j)(y - y_j) + \frac{1}{6}\varphi_{i,xxx}^0 \sum_{j=0}^{M} \varphi_j^0(x - x_j)^3 \\ & + \frac{1}{2}\varphi_{i,xxy}^0 \sum_{j=0}^{M} \varphi_j^0(x - x_j)^2(y - y_j) \\ & + \frac{1}{2}\varphi_{i,xyy}^0 \sum_{j=0}^{M} \varphi_j^0(x - x_j)^1(y - y_j)^2 + \frac{1}{6}\varphi_{i,yyy}^0 \sum_{j=0}^{M} \varphi_j^0(y - y_j)^3 \end{aligned} \tag{15}$$

Then Eq. (8) can be rewritten as

$$u(x, y) = \sum_{i=1}^{M} \Phi_i(x, y)u_i \tag{16}$$

Then, we can see that the present method share several advantages, such as the Kronecker delta function property, high accuracy, high order completeness, lower computational cost and no inversion is needed in the whole process of the shape function construction.

Combining STIM and HBNM, we can obtain the boundary variables interpolation as

$$\tilde{\mathbf{u}} = \begin{Bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{Bmatrix} = \sum_{I=0}^{M} \begin{bmatrix} \Phi_I(x, y) & 0 \\ 0 & \Phi_I(x, y) \end{bmatrix} \begin{Bmatrix} u_1^I \\ u_2^I \end{Bmatrix} \tag{17}$$

$$\tilde{\mathbf{t}} = \begin{Bmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{Bmatrix} = \sum_{I=0}^{M} \begin{bmatrix} \Phi_I(x, y) & 0 \\ 0 & \Phi_I(x, y) \end{bmatrix} \begin{Bmatrix} t_1^I \\ t_2^I \end{Bmatrix} \tag{18}$$

4. Chebyshev polynomials interpolation

According to DRM formulation, the approximation for the inhomogeneous term b_i can be proposed as

$$b_k(x, y) = \sum_{j=1}^{N+L} f^j(r)\alpha_k^j \tag{19}$$

where, α_k^j are a set of unknown coefficients, f^j are basis functions, N and L are total numbers of boundary nodes and internal nodes. It can be seen that the inversion is inevitable for solving unknown coefficients α_k^j, which costs a large amount of computational time.

In the present method, Chebyshev polynomials are employed as the basis function. For simplification, we take two dimensional elasticity problem as the example, then Eq. (9) can be given as

$$b_k(x, y) \approx \sum_{i}^{L} \sum_{j}^{M} a_k^{ij} T^i\left(\frac{2x - x_b - x_a}{x_b - x_a}\right) T^j\left(\frac{2y - y_b - y_a}{y_b - y_a}\right) \tag{20}$$

in which $T^i\left(\frac{2x - x_b - x_a}{x_b - x_a}\right)$ and are Chebyshev polynomials, and the interpolation domain is a rectangle area $[x_a, x_b] \times [y_a, y_b]$, which can cover the calculation domain; Gauss-Lobatto nodes are used as the interpolation nodes; L, M are numbers of Gauss-Lobatto n $T^j\left(\frac{2y - y_b - y_a}{y_b - y_a}\right)$ odes in the x and y directions. So we can see that no real nodes are needed in this interpolation, which overcomes the defect of random arrangement of the traditional radial basis function interpolation.

According to Chebyshev polynomials interpolation theory [31], we can get the interpolation coefficients as

$$a_k^{ij} = \frac{4}{LMc_{L,i}c_{M,j}} \sum_{i_1}^{L} \sum_{j_1}^{M} \frac{b_k(x_{i_1}, y_{j_1})}{c_{L,i_1}c_{M,j_1}} \cos\left(\frac{i\pi i_1}{L}\right)\cos\left(\frac{j\pi j_1}{M}\right) \tag{21}$$

in which $c_{L,0} = c_{L,L} = 2$, and $c_{L,i} = 1$ when $1 \le i \le L-1$, and

$$x_{i_1} = \frac{x_b - x_a}{2}\cos\frac{\pi i_1}{L} + \frac{x_b + x_a}{2} \tag{22}$$

$$y_{j_1} = \frac{y_b - y_a}{2}\cos\frac{\pi j_1}{M} + \frac{y_b + y_a}{2} \tag{23}$$

According to dual reciprocity method theory, the basic form of particular solution must satisfy the following equation

$$G\bar{u}_{mn,ll}^{ij}(x, y) + \frac{G}{1-2\nu}\bar{u}_{ln,lm}^{ij}(x, y) = \delta_{mn}T^i\left(\frac{2x - x_b - x_a}{x_b - x_a}\right)T^j\left(\frac{2y - y_b - y_a}{y_b - y_a}\right) \tag{24}$$

If $\bar{u}_{mn}^{ij}(x, y)$ is known, by means of dual reciprocity method, we can get the particular solution is

$$u_k^p = \sum_{i=0}^{L}\sum_{j=0}^{M} a_l^{ij}\cdot\bar{u}_{lk}^{ij}(x, y) \tag{25}$$

From Eq. (25), interpolation coefficients a_l^{ij} can be referred by Eq. (21), so the next task is solving the basic form of particular solution $\bar{u}_{lk}^{ij}(x, y)$.

In order to solve the basic form of particular solution, the right hand term of Eq. (24) can be rewritten as [31]

$$T^i\left(\frac{2x - x_b - x_a}{x_b - x_a}\right)T^j\left(\frac{2y - y_b - y_a}{y_b - y_a}\right) = \sum_{i_1=0}^{[i/2]}\sum_{j_1=0}^{[j/2]} c_{i_1}^{(i)}c_{j_1}^{(j)}x^{i-2i_1}y^{j-2j_1} \tag{26}$$

in which

$$c_k^{(n)} = (-1)^k 2^{n-2k-1}\frac{n(n-k-1)!}{k!(n-2k)!},\ n > 2k \tag{27}$$

$$c_k^{(2k)} = (-1)^k,\ k \ge 0 \tag{28}$$

Then Eq. (24) can be simplified as

$$G\bar{u}_{mn,ll}(x, y) + \frac{G}{1-2\nu}\bar{u}_{\ln,lm}(x, y) = \delta_{mn}x^{k_1}y^{k_2} \tag{29}$$

in which is shear modulus and ν is Poisson's ratio. Solving Eq. (29), we can get the basic form of particular solution, which can be seen in Table 1. And solving the above Eq. (29), the much more complicate ones of particular solution can be calculated by [8]

Table 1
The basic form of particular solutions.

$p^l(x, y) = x^{k_1}y^{k_2}$	$4G(1-\nu)\bar{u}_{mn}(p^l(x, y))$
1	$\left[2(1-\nu)\delta_{mn} - \delta_{1m}\delta_{1n}\right]x^2$
x^{k_1}	$\left[2(1-\nu)\delta_{mn} - \delta_{1m}\delta_{1n}\right]x^{k_1+2}/C_{k_1+2}^{k_1}$
y^{k_2}	$[2(1-\nu)\delta_{mn} - \delta_{2m}\delta_{2n}]y^{k_2+2}/C_{k_2+2}^{k_2}$
xy	$x^3\{(-\delta_{1m}\delta_{2n} - \delta_{1n}\delta_{2m})x + 4[2(1-\nu)\delta_{mn} - \delta_{1m}\delta_{1n}]y\}/12$
x^2y	$x^4\{(-\delta_{1m}\delta_{2n} - \delta_{1n}\delta_{2m})x + 5[2(1-\nu)\delta_{mn} - \delta_{1m}\delta_{1n}]y\}/30$
xy^2	$y^4\{(-\delta_{1m}\delta_{2n} - \delta_{1n}\delta_{2m})y + 5[2(1-\nu)\delta_{mn} - \delta_{2m}\delta_{2n}]x\}/30$

$$G\bar{u}_{mn} = \frac{1-\nu}{G}H_{mn,kk} - \frac{1}{2G}H_{kn,km} \tag{30}$$

$$\nabla^4 H_{mn} = \delta_{mn}x^{k_1}y^{k_2} \tag{31}$$

From Eqs. (24), (26), (29), we can get that

$$\bar{u}_{mn}^{ij}(x, y) = \sum_{i_1=0}^{[i/2]}\sum_{j_1=0}^{[j/2]} c_{i_1}^{(i)}c_{j_1}^{(j)}\bar{u}_{i_1j_1}(p^l(x, y)) \tag{32}$$

Via Eq. (25), we can get the particular solution of displacement u^p, then applying the equation $\bar{\varepsilon} = (\bar{u}_{mn,l} + \bar{u}_{ml,n})/2$, $\bar{\sigma} = 2G\bar{\varepsilon}_{mnl} + \frac{2G\nu}{1-2\nu}\bar{\varepsilon}_{mjj}\delta_{nl}$, and $\bar{t}_{mn} = \bar{\sigma}_{mnl}n_l$, one can get the basic form of particular solution of traction. The same as Eq. (25), we can get particular solution of traction via the basic form of particular solution of traction, which is

$$t_k^p = \sum_{i=0}^{L}\sum_{j=0}^{M} a_l^{ij}\cdot\bar{t}_{lk}^{ij}(x, y) \tag{33}$$

Eqs. (25) and (33) can be rewritten as matrix form, which are

$$\mathbf{u}^p = \bar{\mathbf{U}}\mathbf{a} \tag{34}$$

$$\mathbf{t}^p = \bar{\mathbf{S}}\mathbf{a} \tag{35}$$

in which $\bar{\mathbf{U}}$ and $\bar{\mathbf{S}}$ are matrixes for basic function of particular solution of displacement and traction respectively, and $\mathbf{a}$ is a matrix of interpolation coefficients of Chebyshev polynomials interpolation. It can be seen that interpolation coefficients can be obtained explicitly by Eq. (21), then no inversion is needed for the particular solution interpolation.

5. Discrete equations of DHBNM

In contrary to the traditional DHBNM, the present method uses STIM to construct the shape function, which has the delta function property, so the boundary condition can be applied easily and directly. From Eqs. (4), (34) and (35), we can get the complementary solution, they can be given in matrix form as

$$\mathbf{u}^c = \mathbf{u} - \mathbf{u}^p = \mathbf{u} - \bar{\mathbf{U}}\mathbf{a} \tag{36}$$

$$\mathbf{t}^c = \mathbf{t} - \mathbf{t}^p = \mathbf{t} - \bar{\mathbf{S}}\mathbf{a} \tag{37}$$

From Eqs. (5) and (6), we can get

$$\mathbf{Tx} = \mathbf{Ht}^c \tag{38}$$

$$\mathbf{Ux} = \mathbf{Hu}^c \tag{39}$$

Substituting Eqs. (36) and (37) into Eqs. (38) and (39), we can get

$$\mathbf{Tx} = \mathbf{H}(\mathbf{t} - \bar{\mathbf{S}}\mathbf{a}) \tag{40}$$

$$\mathbf{Ux} = \mathbf{H}(\mathbf{u} - \bar{\mathbf{U}}\mathbf{a}) \tag{41}$$

Different from the traditional DHBNM, which uses MLS to construct shape function, boundary conditions of the present method can be easily and directly imposed. Appling boundary condition directly and solving linear Eqs. (40) and (41), we can get the solutions of the present problem, and no inversion is needed for both shape function construction and particular solution interpolation.

6. Numerical examples

In order to certify the efficiency of the present new dual reciprocity hybrid boundary node method for solid structure modeling, some numerical examples are considered, which are given as follows. For comparison, we denote the traditional dual reciprocity hybrid boundary node method as DHBNM, and DHRBNM represents the method of dual reciprocity hybrid radial boundary node method in reference [34].

6.1. A quadrifoil shaped plate

A quadrifoil shaped plate is considered in this example [34], in which the geometry of model can be seen in Fig. 1, and the radius of each circle is 1.0. The potential of left and right semicircle is $u = \sin(x/2) + \sin(y/2) + x^3 + y^3$, and normal flux is given as $q = (\cos(x/2)/2 + 3x^2)n_x + (\cos(y/2)/2 + 3y^2)n_y$ on the upper and bottom boundaries, in which n_x and n_y are components of the boundary outside normal direction vector.

For different node numbers, the CPU expenses for this example are given in Fig. 2, in which CPU time for DHBNM with the traditional radial basis function interpolation and DHRBNM with RPIM are also plotted for comparison [34]. It can be seen in this figure that the CPU time expenses are increased slower by the present method than that of by DHBNM and DHRBNM with the increase of the boundary node numbers, and actually no real internal node is needed in the present method. It is obvious that the computational expense is greatly improved by the present method.

The convergences for $e(u)$ of the present method for different node numbers and by different methods are studied in this section, which are shown in Fig. 3. It is shown that the convergence of the present method is much smoother and quicker than that of the DHBNM and DHRBNM [34]. And the equation of $e(u)$ can be referred in reference [34].

Fig. 4 plots the potentials on the internal points on $x = 0.0$ for different methods, which are compared with the results by reference [34]. It is shown that results of the internal points obtained by the present method, DHBNM and DHRBNM are very close with each other, which is demonstrated that the present method is efficient and accurate.

The normal flux errors of the left semi-circle by the different methods are shown in Fig. 5. It can be seen that the most accurate results can be gotten by the present method; besides, the results obtained by the present method are much smoother than the others.

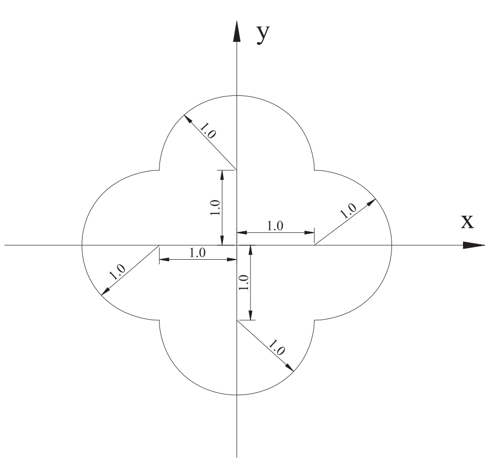

Fig. 1. Geometry of the model.

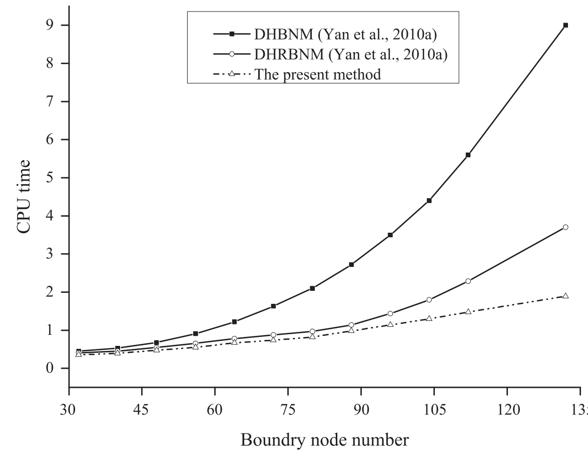

Fig. 2. CPU time expenses for different methods.

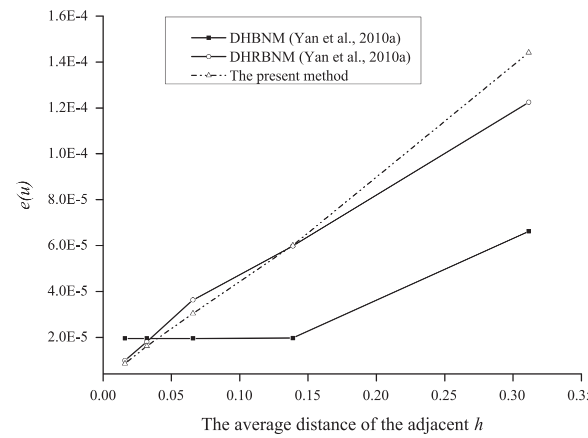

Fig. 3. Convergences for $e(u)$ of the present method compared with DHBNM and DHRBNM.

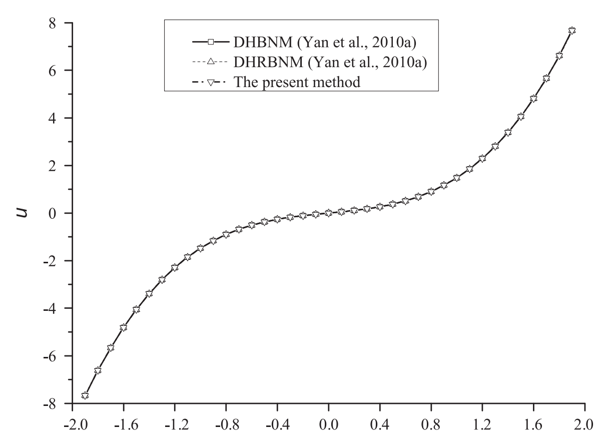

Fig. 4. Potential along the line x=0.0.

6.2. A gravity dam

It is shown in Fig. 6, in which a gravity dam subjected to a hydrostatic pressure on the left side is considered in this example [35]. It is considered as a plane strain problem, and the material properties of the gravity dam are given as $E = 100$ MPa and $\nu = 0.3$. The density of water is 1000 kg/m^3 and the density of gravity dam is 2400 kg/m^3. For comparison, the problem has also been solved by FEM using the commercial package ANSYS, DHBNM and DHRBNM. The displacements at two points A and B (see Fig. 6) are

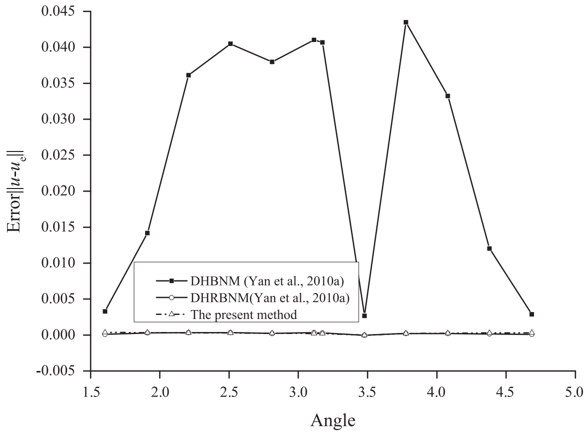

Fig. 5. Flux error by the different methods.

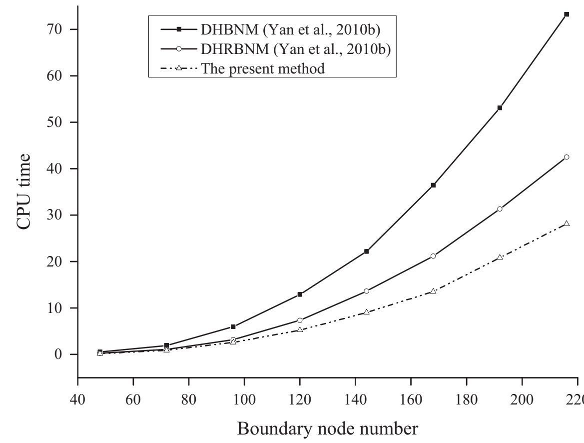

Fig. 7. CPU time expense by the different methods.

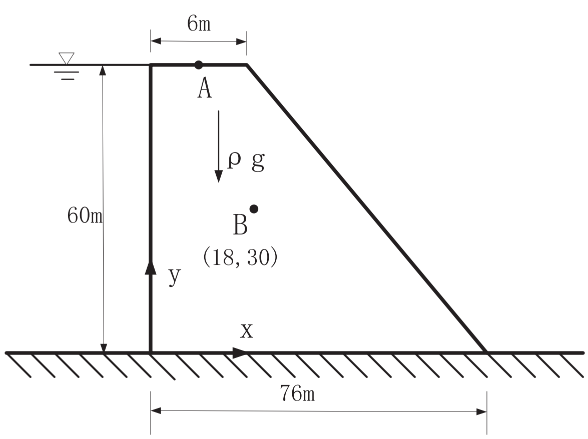

Fig. 6. Geometry of gravity dam.

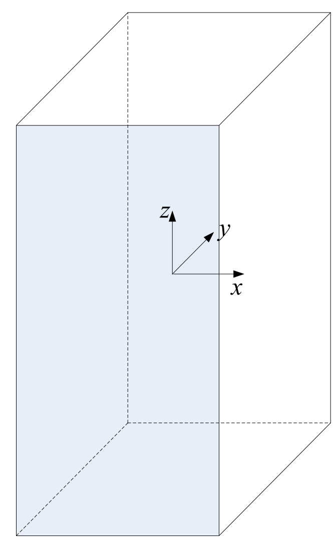

Fig. 8. Model of three dimensional thermal problem.

Table 2
Displacements (m) at A and B of the dam.

Method	Node A (3.0,60.0)		Node B (3.0,60.0)	
	u_x	u_y	u_x	u_y
DHBNM [35]	0.2388	−0.1435	0.1728	−0.1172
DHRBNM [35]	0.2390	−0.1430	0.1739	−0.1174
ANSYS [35]	0.2389	−0.1433	0.1734	−0.1174
The present method	0.2391	−0.1432	0.1737	−0.1175

listed in Table 2. It can be seen that the results obtained by the present method are in good agreement with those obtained by FEM, DHBNM and DHRBNM [35], by DHBNM and DHRBNM some internal nodes are needed, but by the present method no real internal node is needed because of the application of Chebyshev polynomials interpolation.

The CPU time expenses with different boundary node numbers by the different methods are shown in Fig. 7, in which the boundary nodes are increased from 48 to 216, and the internal node number for DHBNM and DHRBNM are the same as reference [35], but no real internal node is needed for the present method. It is obvious that the CPU time of the present method increases slowly with the increase of the boundary node numbers, which is much clearer for the large number of the boundary nodes. It can be observed that the computational expense is greatly improved by the present method.

6.3. Three dimensional thermal load problem

A three dimensional thermal load problem is considered in this section, and the geometry of model can be seen in Fig. 8 [37]. According to thermal theory, the effect of temperature variation on an elastic body is equivalent to that of a pseudo body force and a pseudo surface traction applied on the calculation object. Based on the above, we assume that the temperatures on the faces $x = \pm 0.5$ and $y = \pm 0.5$ are given as $\theta = 100\cos(4x + 4y)$, and the temperatures on the top and bottom surfaces of the model are fixed. A pseudo surface traction of $p_m = -2G(1 + \nu)\alpha\theta n_m/(1 - 2\nu)$ is imposed on faces $x = \pm 0.5$ and $y = \pm 0.5$, and a pseudo body force is applied, which is $b_m = -2G(1 + \nu)\alpha\theta_{,m}/(1 - 2\nu)$. The non-dimensional material parameters are given as $\alpha = 0.00001$, $E = 10000$ and $\nu = 0.3$ [37].

In this case, the exact values of σ_{zz} on the line parallel to the x-axis and $y = 0.0$, $z = -1.0$ are given as $\sigma_{zz} = -100E\alpha\cos(4x + 4y)$, Results by the present method and some other methods are given in Table 3. At the same time, results on the line parallel to the x-axis and $y = 0.0$, $z = -0.5$ and $y = 0.25$, $z = -1.0$ are shown in Fig. 9 and 10. It can be seem that a good agreement can be achieved between the present method, DHBNM and the analytical solution.

Table 3
σ_{zz} on the line of $y = 0.0,\ z = -1.0$.

Coordinate	The present method	DHBNM [37]	Exact [37]
(−0.3,0.0,−1.0)	−3.598	−3.591	−3.624
(−0.2,0.0,−1.0)	−6.962	−6.958	−6.967
(−0.1,0.0,−1.0)	−9.209	−9.205	−9.211
(0.0,0.0,−1.0)	−9.997	−9.994	−10.000

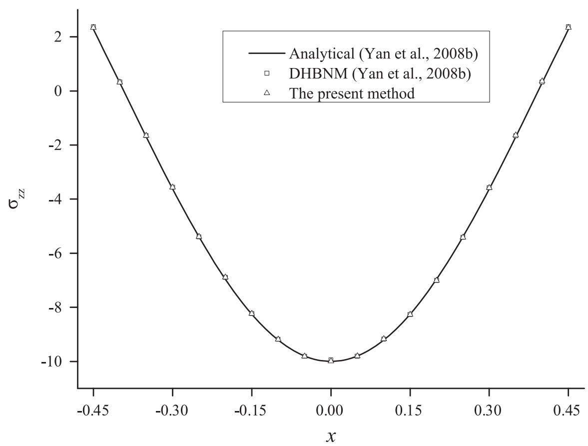

Fig. 9. Distribution of σ_{zz} on the line of $y = 0.0,\ z = -0.5$.

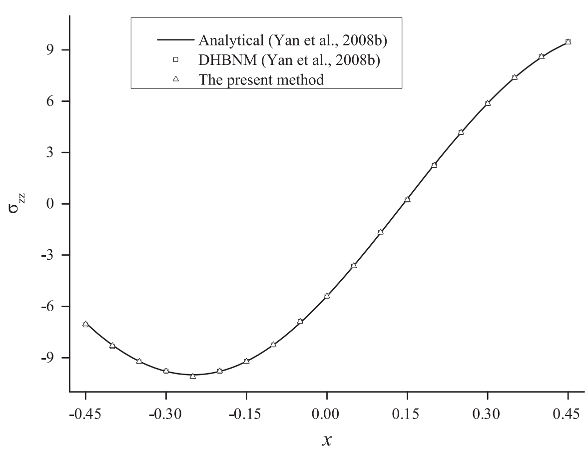

Fig. 10. Distribution of σ_{zz} on the line of $y = 0.25,\ z = -1.0$.

7. Conclusion

In the present paper, a new dual reciprocity hybrid boundary node method is proposed, in which the Shepard and Taylor interpolation method is employed for shape function constructing, and Chebyshev polynomials are applied for interpolating basis functions of particular solution interpolation. Firstly, the Shepard interpolation is used to construct zero level shape function, and the high-power shape functions are constructed through the Taylor expansion, and through those methods, no inversion is needed in the process of the shape function construction, and much lower computational expense is achieved. Besides, Chebyshev polynomials are used as the basis functions of particular solution interpolation instead of the conical function, radial basis functions, and the analytical solution of basic form of particular solutions related to Chebyshev polynomials for elasticity is obtained, by means of Chebyshev polynomials interpolation, no internal interpolation node is needed, and the interpolation coefficients can be given as explicit functions, then no inversion is needed in the process of particular solution interpolation, which costs a large amount of computational expense for the traditional method. Based on those two methods and hybrid boundary node method, a new dual reciprocity hybrid boundary node method is developed, compared to the traditional DHBNM, no inversion is needed for both in the process of shape function construction and particular solution interpolation process, and no internal nodes is needed for particular solution interpolation, which greatly improves the computational efficiency. Numerical examples are given to illustrate that the present method is accurate and effective. And some more challenging practical problems such as: rock fracture, deep underground excavation engineering will be studied by this method, for which a discontinuous method will be developed.

Acknowledgments

The work was financially supported by the National key Basic Research Program of China (No. 2013CB036405), the National Natural Science Foundation of China (No. 41272349 and 51322906), and Youth Innovation Promotion Association CAS (No. 2014304).

References

[1] Atluri SN, Sladek J, Sladek V, Zhu T. The local boundary integral equation and it's meshless implementation for linear elasticity. Comput Mech 2000;25:180–98.
[2] Atluri SN, Zhu T. A new meshless local Petrow-Galerkin approach in computational mechanics. Comput Mech 1998;22:117–27.
[3] Belytschko T, Lu YY, Gu L. Element-free Galerkin methods. Int J Numer Methods Eng 1994;37:229–56.
[4] Boyd JP. Chebyshev and Fourier Spectral Methods. 2nd ed.. New York: Dover; 2001.
[5] Cai YC, Zhu HH. A local meshless Shepard and least square interpolation method based on local weak form. CMES – Comput Model Eng Sci 2008;34:179–204.
[6] Cai YC, Zhu HH. A PU-based meshless Shepard interpolation method satisfying delta property. Eng Anal Bound Elem 2010;34:9–16.
[7] Chen CS, Rashed YF. Evaluation of thin plate spline based particular solutions for Helmholtz-type operators for the DRM. Mech Res Commun 1998;25:195–201.
[8] Cheng AHD, Chen CS, Golberg MA, Rashed YF. BEM for theomoelasticity with body force – a revisit. Eng Anal Bound Elem 2001;25:377–87.
[9] Golberg MA, Chen CS. The theory of radial basis function applied to the BEM for inhomogeneous partial differential equations. Bound Elem Commun 1991;5:57–61.
[10] Golberg MA, Muleshkov AS, Chen CS, Cheng AHD. Polynomial particular solutions for some partial differential operators. Numer Methods Part Differ Equ 2003;19:112–33.
[11] Gu YT, Liu GR. A meshless local Petrov-Galerkin(MLPG) method for free and forced vibration analyses for solids. Comput Mech 2001;27:188–98.
[12] Gu Y, Chen W, Gao HW, Zhang CZ. A meshless singular boundary method for three-dimensional elasticity problems. Int J Numer Methods Eng 2016;107:109–26.
[13] Karageorghis A, Kyza I. Efficient algorithms for approximating particular solutions of elliptic equations using Chebyshev polynomials. Commun Comput Phys 2007;2:501–21.
[14] Lancaster P, Salkauskas K. Surface generated by moving least squares methods. Math Comput 1981;37:141–58.
[15] Lin J, Chen W, Sze KY. A new radial basis function for Helmholtz problems. Eng Anal Bound Elem 2012;36:1923–30.
[16] Liu GR, Yan L, Wang JG, Gu YT. Point interpolation method based on local residual formulation using raidal basis functions. Struct Eng Mech 2002;14:713–32.
[17] Mason JC, Handscomb DC. Chebyshev Polynomials. Boca Raton: Chapman & Hall/CRC; 2003.
[18] Melenk JM, Babuska I. The partition of unity finite element method: basic theory and applications. Comput Methods Appl Mech Eng 1996;139:289–314.
[19] Miao Y, Wang YH, Jiang HY. An improved hybrid boundary node method in two-dimensional solids. Acta Mech Solid Sin 2005;18:307–15.
[20] Miao Y, Wang YH. Meshless analysis for three-dimensional elasticity with singular hybrid boundary node method. Appl Math Mech 2006;27:673–81.
[21] Miao Y, Sun TC, Zhu HP, Wang Q. A new model for the analysis of reinforced concrete members with a coupled HdBNM/FEM. Appl Math Model 2014;38:5582–91.

[22] Miao Y, Chen Z, Wang Q, Zhu HP. Mechanical analysis of 3D composite materials by hybrid boundary node method. CMC: Comput Mater Contin, 43; 2014. p. 49–73.
[23] Miao Y, Wang Q, Zhu HP. Thermal analysis of 3D composites by a new fast multipole hybrid boundary node method. Comput Mech 2014;53:77–90.
[24] Mukherjee YX, Mukherjee S. The boundary node method for potential problems. Int J Numer Methods Eng 1994;40:797–815.
[25] Muleshkow AS, Golberg MA, Chen CS. Particular solutions of Helmholtz-type operators using higher order polyharmonic splines. Comput Mech 1999;23:411–9.
[26] Nardini D, Brebbia CA. A new approach to free vibration analysis using boundary elements boundary element methods in engineering. Southampton: computational mechanics publications.Berlin and New York: Springer; 1982.
[27] Nardini D, Brebbia CA. Transient dynamic analysis by the boundary element method boundary element methods in engineering. Southampton: computational mechanics publications.Berlin and New York: Springer; 1983.
[28] Partridge PW, Brebbia CA, Wrobel LC. The dual reciprocity boundary element method.London/New York: CMP/Elsevier; 1992.
[29] Reutskiy SY, Chen CS. Approximation of multivariate functions and evaluation of particular solutions using Chebyshev polynomial and trigonometrix basis functions. Int J Numer Methods Eng 2006;67:1811–29.
[30] Tinh QB, Tan NN, Hung ND. A moving Kriging interpolation-based meshless method for numerical simulation of Kirchhoff plate problems. Int J Numer Methods Eng 2009;77:1371–95.
[31] Tsai CC. The particular solutions of Chebyshev polynomials for Reissner plates under arbitrary loadings. CMES – Comput Model Eng Sci 2009;45:249–71.
[32] Wrobel LC, Brebbia CA. The dual reciprocity boundary element formulation for non-linear diffusion problems. Comput Methods Appl Mech Eng 1987;65:147–64.
[33] Yan F, Feng XT, Lv JH, Pan PZ. A new hybrid boundary node method based on Taylor expansion and Shepard interpolation method. Int J Numer Methods Eng 2015;102:1488–506.
[34] Yan F, Feng XT, Zhou H. A dual reciprocity hybrid radial boundary node method based on radial point interpolation method. Comput Mech 2010;45:541–52.
[35] Yan F, Feng XT, Zhou H. Meshless method of dual reciprocity hybrid radial boundary node method for elasticity. Acta Mech Solid Sin 2010;23:447–58.
[36] Yan F, Wang YH, Tham LG, Cheung YK. Dual reciprocity hybrid boundary node method for 2-D elasticity with body force. Eng Anal Bound Elem 2008;32:713–25.
[37] Yan F, Wang YH, Miao Y, Tan F. Dual reciprocity hybrid boundary node method for three-dimensional elasticity with body force. Acta Mech Solid Sin 2008;21:267–77.
[38] Yan F, Wang YH, Miao Y, Tham LG, Cheung YK. Dual reciprocity hybrid boundary node method for free vibration analysis. J Sound Vib 2009;321:1036–57.
[39] Yang FL, Chen CH, Young DL. A novel mesh regeneration algorithm for 2D FEM simulations of flows with moving boundary. J Comput Phys 2011;230:3276–301.
[40] Zhang JM, Yao ZH, Li H. A Hybrid boundary node method. Int J Numer Methods Eng 2002;53:751–63.
[41] Zhang JM, Yao ZH, Masataka T. The meshless regular hybrid boundary node method for 2-D linear elasticity. Eng Anal Bound Elem 2003;127:259–68.
[42] Zhang X, Song KZ, Lu MW, Liu X. Meshless methods based on collocation with radial basis functions. Comput Mech 2000;26:333–43.
[43] Zhu T, Zhang JD, Atluri SN. A local boundary integral equation(LBIE) method in computational mechanics, and a meshless discretization approach. Comput Mech 1998;21:223–35.