Determination of Hydraulic Properties of Unsaturated Soils Based on Nonequilibrium Multistep Outflow Experiments

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Abstract: In this paper, a new procedure for rapidly determining the hydraulic properties of unsaturated soils is developed. The proposed procedure is used to characterize the evolution of the instant degree of saturation in an unsaturated soil during a multistep outflow experiment. By introducing the concept of capillary relaxation, a theoretical model is developed and adopted to describe the dynamic hydraulic properties of unsaturated soils under constant-volume conditions. A series of nonequilibrium multistep outflow experiments are performed on various types of unsaturated soils. It is shown that the evolution of the instant degree of saturation or the outflow mass generally follows an exponential law. Both experimental results and theoretical simulations illustrate that flow instability and water blockage can occur in the soil during multistep outflow experiments. The theoretical predictions are compared to the experimental results, showing that the proposed procedure can be effectively used to determine the hydraulic properties of unsaturated soils. Because the outflow test is performed under nonequilibrium flow conditions, the experimental time for measuring the hydraulic properties is much shorter than those needed for conventional equilibrium procedures. **DOI:** 10.1061/(ASCE)GT.1943-5606.0001598.

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Introduction

Determination of the hydraulic properties of unsaturated soils is a crucial part in analyzing many subsurface hydraulic and mechanical processes, such as flow and transport in the vadose zone; precipitation-induced landslides, subsidence due to groundwater extraction, groundwater management, and so on (Morse et al. 2014; Ranalli et al. 2014). These properties include the soil-water characteristic curve (SWCC) and the hydraulic conductivity function (HCF) of the soils. The former is represented by a relationship between matric suction and degree of saturation, while the latter describes the dependence of conductivity on the degree of saturation or the matric suction. The hydraulic properties of an unsaturated soil can be experimentally determined in either direct or indirect ways (Lu and Likos 2004), or numerically inferred from a back-analysis of some initial/boundary value problem with incorporation of an optimization algorithm (Wayllace and Lu 2012).

Traditionally, experimental measurements of hydraulic properties of unsaturated soils are conducted under equilibrium conditions. In these measurements, the moisture content at every applied matric suction step is measured only when the matric suction (usually the negative pore-water pressure) becomes equilibrated over the whole soil sample. It is clear that such measurements are generally time-consuming, since a certain amount of time is required for the pore pressure to become equilibrated under every suction step applied. This is the case, especially for fine-grained soils or at low saturation where the hydraulic conductivity is small.

In the numerical determination of hydraulic properties, a one-step (OSO) or multistep (MSO) outflow test is first conducted, and by treating the flow process in the soil sample as an initial/boundary value problem, the hydraulic properties of the soil can be determined by numerically solving the problem using a back-analysis procedure with a proper optimization algorithm. Compared to the experimental measurements, the back-analysis procedures represent more-rapid techniques for determining the hydraulic properties of unsaturated soils. For the OSO-based back-analysis, however, the estimate of the hydraulic constitutive functions is not unique, since the cumulative water outflow depends on the magnitude of the applied matric suction (Wildenschild et al. 2001). In contrast, for the MSO-based back-analysis, it has been concluded that use of cumulative water outflow in the objective function leads to unique hydraulic constitutive functions (van Dam et al. 1994; Hopmans et al. 2002). It has been recognized, however, that if the MSO-based analysis is performed, equilibrium in the simulated cumulative outflow is achieved much faster than experimentally observed (Schultz et al. 1999; Hwang and Powers 2003). The discrepancy between observed and fit behavior can be attributed to the nonequilibrium effect of matric suction (or alternatively, capillary pressure) (Barrenblatt and Gil'man 1987; Hassanizadeh and Gray 1990; Das and Mirzaei 2012).

The physical processes that induce the nonequilibrium effect are not well understood. Thus far, several mechanisms have been proposed to explain the occurrence of such a nonequilibrium phenomenon (Kalaydjian 1992; Wildenschild et al. 2001;

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Hassanizadeh et al. 2002; Wei and Muraleetharan 2007). When an advancing liquid (say pore water) invades a porous medium, the curvature of the wetting/nonwetting interface is unable to change smoothly according to the matric suction change. Under these circumstances, the interface becomes unstable and jumps from one stable location to a new stable location (i.e., the Haines jump). Such pore-scale processes cannot be captured when upscaling from the pore to a representative elementary volume (REV) scale. Due to these unstable processes, however, as the advancing liquid invades the porous medium, the measured matric suction will be larger and will decrease when the pore fluids reach equilibrium, resulting in the nonequilibrium effect in matric suction (Kalaydjian 1992; Nikooee et al. 2013). It was hypothesized that other mechanisms such as water or air entrapment, pore-water blockage, air-entry effect, and dynamic contact angle effect, could also contribute to the nonequilibrium effect in matric suction (Wildenschild et al. 2001). Following a detailed review of relevant experimental studies, Hassanizadeh et al. (2002) suggested that the nonequilibrium effect is related to the presence of local heterogeneities in the porous medium, which are not properly captured by continuum hydraulic models. Recently, based on an analysis of the acoustical behavior of partially saturated porous media, Wei and Muraleetharan (2007) pointed out that due to existence of local heterogeneities, local distribution of pore pressure in a porous medium is not uniform, and such nonuniformity of pore pressure can induce nonequilibrium effect in matric suction, resulting in local fluid flow.

Based on the thermodynamic theory of porous media, Hassanizadeh and Gray (1990) have developed a model to account for the nonequilibrium effect in capillary pressure. This model suggests that the nonequilibrium (or dynamic) capillary pressure (p_c^{dyn}) has two contributions: (1) capillary pressure at equilibrium (p_c^{eq}), and (2) a linear function of the rate of saturation ($\dot{S}_r$), e.g., $p_c^{dyn} = p_c^{eq} - \xi \dot{S}_r$, where ξ is a material coefficient, and p_c^{eq} is a function of the degree of saturation (S_r). Indeed, the function $p_c^{eq}(S_r)$ simply represents the soil-water characteristic curve (SWCC) measured under equilibrium conditions. This dynamic model has been implemented into a MSO-based back-analysis procedure to estimate the hydraulic properties of unsaturated porous media (O'Carroll et al. 2005). It is shown that, because the mentioned dynamic model of capillary pressure was utilized, there was significant improvement in the agreement between measured and simulated cumulative water outflow and the outflow rate, and the procedure yielded reasonably good estimate of the hydraulic properties.

Although Hassanizadeh and Gray's model accounts for the nonequilibrium effect in capillary pressure, its material coefficient (ξ) depends on both saturation and flow rate. Due to this feature, a proper functional form for ξ has to be presumed so that the dynamic mode can be effectively used to simulate the nonequilibrium effect. A close examination of O'Carroll et al.'s (2005) results reveals that if a constant ξ is assumed, significant discrepancy still remains between the measured and the simulated cumulative outflows. To resolve this issue, Wei and Dewoolkar (2006) suggested that a linear form of Hassanizadeh and Gray's (1990) model should be used to simulate the nonequilibrium effect occurring in MSO experiments. If the size of matric suction step is small enough, it is reasonable to assume that ξ is constant during each suction step, and for a nonequilibrium MSO experiment, a closed-form expression can be developed for the variation of saturation with the applied matric suction.

In this paper, the linear form of Hassanizadeh and Gray's (1990) model is employed to simulate nonequilibrium MSO experiments, and a new procedure is developed to effectively determine the hydraulic properties of unsaturated soils. The proposed procedure distinguishes itself from other methods for determining the hydraulic properties of unsaturated soils, in that it avoids numerically solving an initial/boundary value and it requires only nonequilibrium outflow testing data. Hence, compared to other methods, the new procedure is much simpler and more effective with regard to the time required to determine the hydraulic properties.

Theory

Linear Dynamic Model of Soil-Water Characteristics

During a hydraulic process, the instant matric suction (or nonequilibrium capillary pressure) in an unsaturated soil can be generally represented by (e.g., Hassanizadeh and Gray 1990; Hassanizadeh et al. 2002; Wei and Muraleetharan 2007)

$$p_c^{dyn} = p_c^{eq} - \xi \dot{S}_r \tag{1}$$

where p_c^{dyn} = instant, nonequilibrium matric suction; p_c^{eq} = matric suction at equilibrium; S_r = instant degree of saturation; and ξ = positive material coefficient. In practice, p_c^{eq} is a function of S_r only, and function $p_c^{eq}(S_r)$ represents the conventional SWCC. Eq. (1) implies that the instant matric suction has two contributions, i.e., an equilibrium part, p_c^{eq}, and a nonequilibrium part, depending upon the rate of the instant degree of saturation. The attribute instant is used here to distinguish $S_r(t)$ from its equilibrium counterpart, and to highlight the fact that in a transient hydraulic process the pore-water pressure and the moisture could be heterogeneously distributed over the pore space in the soil.

To shed insight into the physics of Eq. (1), consider a special case where a representative volume of unsaturated soil is disturbed by a small matric suction change, δp_c. Assume that the initial matric suction and the initial degree of saturation are p_{c0} and S_{r0}, respectively. After δp_c is applied, the instant change of the degree of saturation is $\delta S_r(t)$. Here, δS_r varies with time after δp_c is applied, since the moisture will locally redistribute in the soil due to the suction change, which requires a certain amount of time. This process is called capillary relaxation (Wei and Muraleetharan 2007). With Eq. (1), one has

$$p_{c0} + \delta p_c = p_c^{eq}(S_{r0} + \delta S_r) - \xi \dot{S}_{r0} - \xi \delta \dot{S}_r \tag{2}$$

As S_{r0} is constant and $p_c^{eq}(S_{r0}) = p_{c0}$, the preceding equation can be linearized to yield

$$\delta p_c = \left.\frac{dp_c^{eq}}{dS_r}\right|_{S_{r0}} \delta S_r - \xi \delta \dot{S}_r + O(\delta S_r) \tag{3}$$

where $O(\delta S_r)$ = sum of the higher-order terms of δS_r. One can define a parameter C such that

$$\frac{1}{C} = \left.\frac{dp_c^{eq}}{dS_r}\right|_{S_{r0}} \tag{4}$$

Clearly, C is a function of S_{r0} (or equivalently, p_{c0}) and represents the water storage capacity of the soil. If δp_c is small enough, the higher-order term can be dropped from Eq. (3), and one can cast Eq. (3) into

$$\delta p_c = \frac{1}{C}\delta S_r + \frac{\tau}{C}\delta \dot{S}_r \tag{5}$$

where $\tau(= -C\xi)$ = characteristic time of capillary relaxation, which is also a function of S_{r0} (or equivalently, p_{c0}). Noticeably, parameters τ and C remain practically unchanged during a small suction step, δp_c.

Variation of Saturation in OSO and MSO Experiments

Now consider a small representative volume of the unsaturated soil. For later experimental justification, it is assumed that the volume has a size and a shape similar to those of the sample used in the measurement of a SWCC. Based on the axis-translation technique, the matric suction is applied by varying the air pressure on the top of the sample while keeping the pore-water pressure at the bottom constant. When a small change in matric suction is applied to the top of the sample, the water will be squeezed out of or sucked into the sample through the bottom until the matric suction become equilibrated over the whole sample. Supposed that the soil is initially at equilibrium with initial saturation and matric suction of S_{r0} and p_{c0}, respectively. Then a stepwise matric suction change is applied, i.e., $\delta p_c(t) = \Delta p_c H(t)$, where Δp_c is a small constant and $H(t)$ is the Heaviside function. Now Eq. (5) can be solved for

$$S_r(t) = S_{r0} + \Delta p_c C\left[1.0 - \exp\left(-\frac{t}{\tau}\right)\right] \tag{6}$$

Physically, Eq. (6) implies that after a step of matric suction is applied, the total change in the saturation is not immediately completed, and instead the process will take a certain amount of time for the generated nonuniform pore-water pressure to fully dissipate so that the whole system arrives at a new equilibrium state, at which the final saturation equals ($S_{r0} + \Delta p_c C$). It is clear that the duration of the described process is accounted for by parameter τ, which in turn depends on the hydraulic conductivity of the soil (Wei and Muraleetharan 2007). Indeed, as shown later, τ stores the information that can be used to infer the hydraulic conductivity.

In Eq. (6), parameters τ and C have yet to be determined. To this end, one can perform a one-step outflow experiment to measure the variation of outflow with time. Based on the measured outflow data, the temporal variation of saturation can be calculated, provided that the physical properties of the soil including the porosity (or void ratio) and the initial saturation are predetermined. By matching the simulated and the measured temporal variations of saturation, parameters τ and C can be determined using a simple trial-and-error method.

In the MSO experiment, the applied matric suction can be described by

$$\delta p_c(t) = \sum_{i=1}^{N} \Delta p_c^i H(t - t_i) \tag{7}$$

where N = total number of matric suction steps applied in the experiment; and Δp_c^i = magnitude of the matric suction change at step i, which is applied at moment t_i. With $\delta p_c(t)$ defined by Eq. (7), one can solve Eq. (5) for

$$S_r(t) = S_{r0} + \sum_{i=1}^{N} \Delta p_c^i C_i\left[1.0 - \exp\left(-\frac{t - t_i}{\tau_i}\right)\right] \tag{8}$$

where τ_i and C_i = values of τ and C at step i, respectively, which can be determined using the results of a multistep outflow experiment, as described in the following section. As such, one obtains a one-to-one relationship between τ_i (or C_i) and the applied matric suction, $p_c^i = p_{c0} + \sum_{k=1}^{i} \Delta p_c^k$. Clearly, Eq. (8) accounts for the effect of heredity. That is, in a nonequilibrium MSO experiment, the incomplete variation of saturation in a previous nonequilibrium step will accumulate into the subsequent steps.

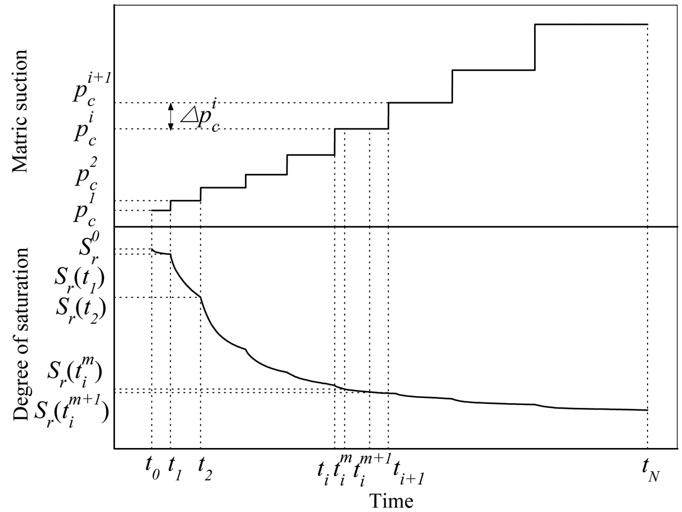

Fig. 1. Temporal variations of matric suction and instant saturation in a MSO experiment

Hydraulic Properties Determined by MSO Experiments

To determine the SWCC and conductivity function based on the proposed model, one has to establish the relationships of τ_i (and C_i) versus p_{ci}. For this purpose, a MSO experiment has to be conducted. From the MSO experiment, one can determine the temporal variation of saturation in the sample, as shown in Fig. 1.

Consider step i. On the time-history curve of saturation, the authors arbitrarily choose M representative points, at which the saturations and corresponding times can be directly read as $(S_{r,i}^1, t_i^1)$, $(S_{r,i}^2, t_i^2)$, ..., $(S_{r,i}^M, t_i^M)$, respectively. By Eq. (8), one can calculate that

$$S_r(t_i^m) = S_{r0} + \sum_{k=1}^{i} \Delta p_c^k C_k\left[1.0 - \exp\left(-\frac{t_i^m - t_k}{\tau_k}\right)\right] \tag{9}$$

where $m = 1, 2, \ldots, M$. Supposed that τ_k and C_k ($k \leq i - 1$) have been determined. Then on the right-hand side of Eq. (9), the only unknowns are τ_i and C_i. Hence $S_r(t_i^m)$ can be expressed as a function of t_i^m, τ_i, and C_i, denoted by $S_r(t_i^m, C_i, \tau_i)$. To calculate τ_i and C_i, one can use the least-square method. The problem now is stated as one of finding τ_i and C_i, so that function $E(C_i, \tau_i)$ achieves its minimum, where

$$E(C_i, \tau_i) = \sum_{m=1}^{M} [S_{r,i}^m - S_r(t_i^m, C_i, \tau_i)]^2 \tag{10}$$

Starting from the first step ($i = 1$), one can repeatedly use the described procedure to determine all τ_i and C_i ($i = 1, 2, \ldots, N$).

At step i, the total matric suction applied on the sample is

$$p_c^i = p_{c0} + \sum_{k=1}^{i} \Delta p_c^k \tag{11}$$

and the corresponding degree of saturation at equilibrium is given by

$$S_{ri}^{eq} = \lim_{t\to\infty}\left\{S_r^0 + \sum_{k=1}^{i}\Delta p_c^k C_k\left[1.0 - \exp\left(-\frac{t-t_k}{\tau_k}\right)\right]\right\}$$
$$= S_r^0 + \sum_{k=1}^{i}\Delta p_c^k C_k \tag{12}$$

As such, the authors have established a one-to-one relationship between p_c^i and S_{ri}^{eq} at equilibrium ($i = 1, 2, \ldots, N$). If the magnitudes of the applied suction step are reasonably small, Eq. (12) should yield a good estimate for the SWCC of the unsaturated soil under equilibrium conditions.

To determine the conductivity function, the authors assume that the volumetric change of the soil during the MSO experiment is negligible and the pore water is incompressible. In this case the mass balance equation for pore water is given by

$$n\frac{\partial S_r}{\partial t} + \nabla\cdot(nS_r\mathbf{v}^w) = 0 \tag{13}$$

where $\mathbf{v}^w$ = velocity of water, given by a Darcy's type flow equation, i.e.,

$$nS_r\mathbf{v}^w = -\frac{k_0 k_r}{\gamma_w}\nabla p_w \tag{14}$$

where γ_w i = unit weight of the pore water; p_w = pore-water pressure; n = porosity of the soil; k_0 [L/T] = conductivity of the soil at full saturation; and $k_r \in [0,1]$ = relative conductivity function, which depends on the saturation or equivalently on the matric suction.

Eqs. (13) and (14) are now applied to simulate the seepage process during a MSO experiment. It is assumed that during each suction step, k_r is constant and depends only upon the current matric suction. For step i, it can be derived from Eq. (4) that

$$\frac{\partial S_r}{\partial t} = C_i\frac{\partial p_c}{\partial t} \tag{15}$$

With the preceding conditions, by inserting Eq. (14) into Eq. (13), one obtains

$$nC_i\frac{\partial p_c}{\partial t} = -\frac{k_0 k_r}{\gamma_w}\nabla\cdot\nabla p_c \tag{16}$$

Here it is assumed that the pore air pressure is constant through the whole sample so that $\delta p_w = -\delta p_c = -\Delta p_c^i$.

In the MSO experiment, the one-dimensional (1D) flow condition is assumed in the sample (Fig. 2). Let t_i^{eq} be the equilibrium time for step i. Then t_i^{eq} can be determined using parameters τ_i and C_i, and the procedure for determining t_i^{eq} is given in the Appendix. Now from Eq. (16), it follows that for the 1D flow

$$nC_i\frac{\Delta p_c^i}{t_i^{eq}} \approx -\frac{k_0 k_r^i}{\gamma_w}\frac{\Delta p_c^i}{H^2} \tag{17}$$

where H = length of seepage path, i.e., the height of the sample; and k_r^i = relative conductivity at step i. Finally, one obtains that

$$k_0 k_r^i \approx -n\gamma_w C_i\frac{H^2}{t_i^{eq}} \tag{18}$$

Because both C_i and t_i^{eq} depend only on p_c^i or S_r^i [Eqs. (11) and (12)], $k_0 k_r^i$ is a function of p_c^i or S_r^i. This function represents the conductivity function of the unsaturated soil. If k_0 is determined, Eq. (18) can be used to obtain the relative conductivity function k_r^i.

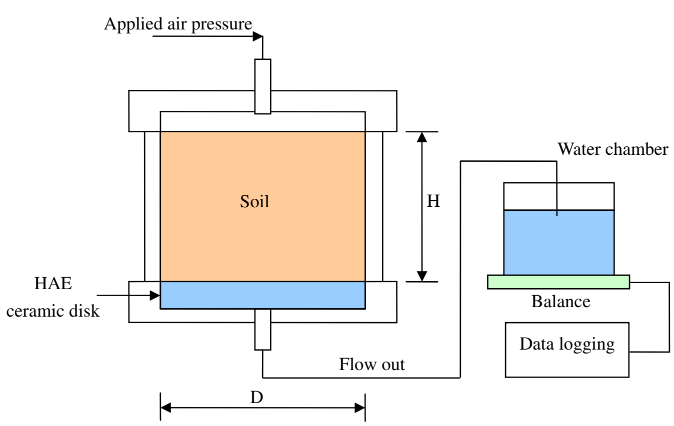

Fig. 2. Schematic of the outflow experiment

Determination of Hydraulic Functions

Nonequilibrium MSO Tests

Multistep outflow experiments were conducted using a modified version of the transient water release and imbibitions method (TRIM) system originally developed at the Colorado School of Mines (Lu et al. 2006). The setup of the TRIM system is schematically shown in Fig. 2. A cylindrical specimen (diameter $D = 5.4$ cm, height $H = 3.0$ cm) is placed in an acrylic confining cell with a high air-entry value (HAE) ceramic disk placed in good contact on the bottom. The top of the cell is connected to an external source of air pressure, while the bottom is connected, via a water tube, to a water chamber, in which the water level is approximately at the same height as the middle plane of the specimen. In outflow experiments, the air pressure is changed on the top of the sample, and the water is expelled out of or sipped into the sample through the HAE ceramic disk on the bottom. The outflow mass is determined by measuring the variation of the weight of the water chamber. Provided that the initial water content and porosity of the specimen is known, the variation of the water content can be determined using the measured outflow mass.

A series of MSO experiments were performed on four types of soils with different compositions, including a silty sand, a silt, a silty clay, and a clay, whose classifications and physical properties are given in Table 1, respectively. The tested samples were statically compacted to the targeted dry densities. For comparison, the SWCC was independently determined for each soil using a pressure plate extractor (Soilmoisture Equipment, Goleta, California). The saturated hydraulic conductivity of each soil was also determined by an independent flow test using the TRIM system. The air entry values of the soils were obtained using the TRIM system, in a way similar to the method suggested in ASTM D6838-02 (ASTM 2008). Before a MSO test, an increment of matric suction (approximately equal to the air entry value) was applied to the sample until the soil reached equilibrium, and the initial moisture content and matric suction were determined. Then, a series of incremental suction steps, each of which had a specified duration time, were applied to the sample by increasing the air pressure on the top. The outflow water mass was recorded by a balance and data-logging system. The duration time of each suction step was chosen in such a way that it was much shorter than the equilibration time but still long enough to allow the measured curve of outflow versus time to become saliently a relaxation-type curve. The authors shall further discuss this point later.

Table 1. Classification and Physical Properties of Tested Soils

Properties	Tested soils			
	Silty sand	Silt	Silty clay	Clay
USCS classification	SM	ML	CL	CH
Specific gravity, G_s	2.70	2.71	2.72	2.73
Dry density, ρ_d (g/cm^3)	1.64	1.72	1.68	1.29
Porosity, n	0.393	0.365	0.382	0.527
Grain composition (%) <0.002 mm	3.9	4.7	39.5	65.3
Grain composition (%) 0.002–0.075 mm	39.6	76.8	45.5	26.6
Grain composition (%) 0.075–0.1 mm	56.5	18.5	15.0	8.1
Liquid limit (%)	—	33.5	38.3	56.9
Plastic limit (%)	—	29.4	21.6	27.0
Plasticity index (%)	—	9.6	16.6	29.9
Saturated hydraulic conductivity (cm/s)	2.7×10^{-4}	1.1×10^{-5}	7.8×10^{-6}	3.1×10^{-6}
Air entry value (kPa)	10.0	20.0	15.0	20.0

Determination of SWCC and HCF

Parameters τ_i and C_i are obtained by solving Eqs. (9) and (10) at each matric suction, and the solution for the first step is used as the input in the second step, until all the solutions for each step are obtained. The degree of saturation at equilibrium is determined by Eq. (12). As such, one can obtain a series of data pairs (p_c^i, S_r^i), from which one obtains the SWCC at equilibrium.

The hydraulic conductivity function (HCF) can be obtained using Eq. (18). To validate the proposed method, the predicted hydraulic conductivity function is compared to the results from that determined by an empirical model. For this purpose, the empirical model of the hydraulic conductivity function by Mualem (1976) and van Genuchten (1980) is adopted, i.e.,

$$k_r = (S_e)^{0.5}\{1 - [1 - (S_e)^{\frac{n}{n-1}}]^{\frac{n-1}{n}}\}^2 \qquad (19a)$$

$$S_e = \frac{S_r - S_{\mathrm{irr}}}{1 - S_{\mathrm{irr}}} = [1 + (\alpha P_c)^n]^{-\frac{n-1}{n}} \qquad (19b)$$

where k_r = relative hydraulic conductivity function of water; S_e = effective degree of saturation of water; S_{irr} = residual degree of saturation of water; and α and n = empirical parameters, which can be determined by fitting the measured SWCC at equilibrium.

Results and Discussions

Figs. 3–5 illustrate the experimental results and theoretical simulations of the nonequilibrium MSO experiments for the silty sand. Three schemes were adopted to apply the matric suction steps, which are denoted by Tests 1, 2 and 3, respectively, as shown in Fig. 3. The temporal variations of the instant degree of saturation, i.e., $S_r(t)$, are calculated from the temporal variation of outflow water mass, which can be accurately measured in the outflow experiment.

As expected, the variation of the instant degree of saturation (or equivalently, the outflow mass) generally lags behind the applied matric suction. At the first step, the instant saturations are quite different for different suction application schemes, though the magnitudes of the first suction step are the same (10 kPa). Particularly, in Test 3, where the duration of the first step is longest, $S_r(t)$ is significantly lower (i.e., the outflow mass is larger) than those in the other two. Clearly, both the duration and the size of the applied suction step can influence the induced outflow mass. Other than the duration and step size, however, some other factors can also contribute to the outflow process. For example, during the second step, $S_r(t)$ decreases much faster in Test 2 than in Test 1, even though the magnitudes of the applied suction step are the same. This cannot be explained by the difference of step durations, since the temporal variation curves in both tests diverge much earlier than the end of Test 3, which has a shorter duration. The discrepancy can be

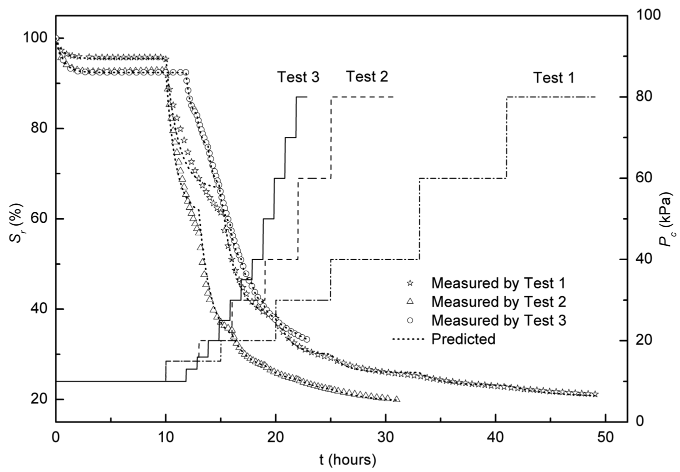

Fig. 3. Experimental schemes and results of the nonequilibrium MSO experiment on the silty sand

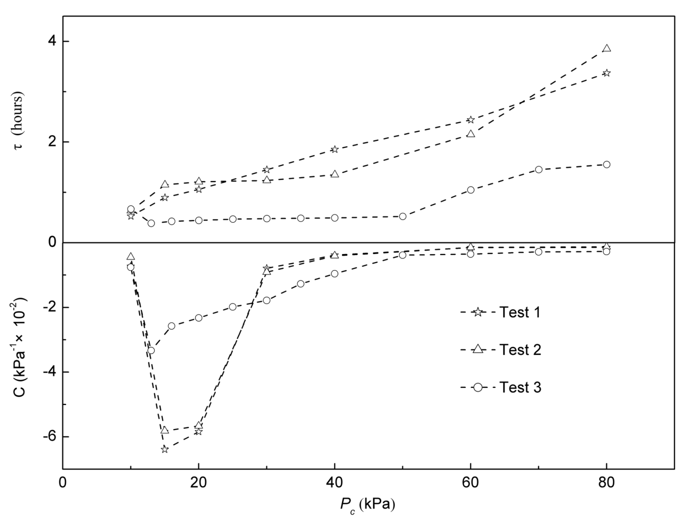

Fig. 4. Variations of parameters C and τ with applied matric suction for the silty sand

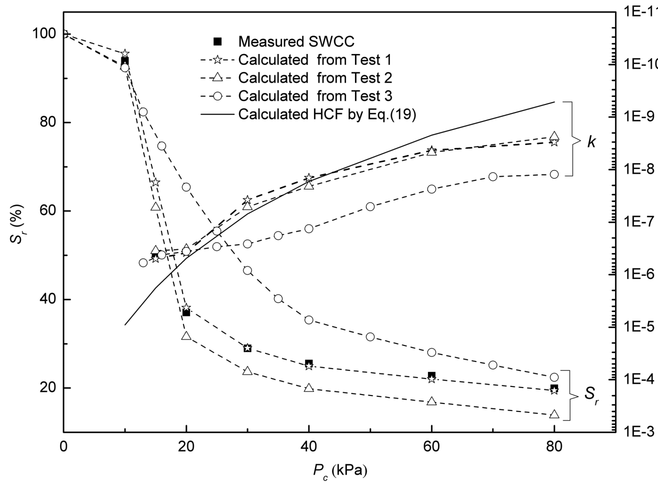

Fig. 5. Comparison of the calculated and measured hydraulic properties for the silty sand

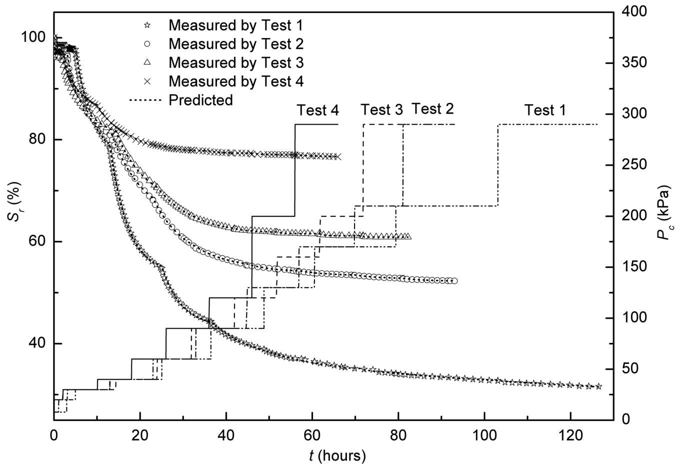

Fig. 6. Experimental schemes and results of the nonequilibrium MSO experiment on the silt

attributed to the instability occurring in the drying process. Here, for a sandy soil, in the vicinity of the air entry value, a small increment of matric suction can drive a significant amount of pore water out of the soil, resulting an apparent instability phenomenon (Kalaydjian 1992). This is probably the main reason that the outflow mass variations are significantly different among the three tests during the first two steps, where the matric suction varies in the vicinity of the air entry value (~10 kPa).

The predicted instant saturations are given in Fig. 3 for comparison. Clearly, the predictions agree very well with the measurements, except for Step 2, where instable flow might have occurred. This result implies that the moisture redistribution process in the MSO experiment follows an exponential law, which can be described by Eq. (6) for a single step or Eq. (9) for multiple steps. It is because of this feature that the local moisture redistributing process in the unsaturated soil is simply termed as capillary relaxation (Wei and Muraleetharan 2007).

The variations of τ_i and C_i with the applied matric suction are shown in Fig. 4. The SWCCs and the HCFs can be calculated using τ_i and C_i, as discussed in section "Hydraulic Properties Determined by MSO Experiments," and the results are presented in Fig. 5, where the SWCC measured at equilibrium and the HCF inferred from Eqs. (19*a*) and (19*b*) are also given for the purpose of comparison. It can be seen that the calculated SWCC based on the outflow measurement of Test 1 agrees very well with the SWCC measured at equilibrium, while the calculated HCFs based on the measured data of Test 1 agree generally well with that inferred from Eqs. (19*a*) and (19*b*). In addition, the calculated SWCC based on the results of Test 2 is similar to the measured result, though the saturation is slightly underestimated at the same matric suction, and the calculated HCF also agrees reasonably well with that inferred from Eqs. (19*a*) and (19*b*). The discrepancy probably stems from the instability occurring in the first two steps of applied matric suction, as illustrated by the variations of the measured outflow water mass in Fig. 3 (compare Test 2 and Test 1)

In a sharp contrast, however, based on Test 3 where the rate of the applied matric suction is the fastest, both the calculated SWCC and HCF deviate significantly from those for Tests 1 and 2. The discrepancy can be attributed to the water blockage effect as well as the instability occurring in the first several suction steps. The pore-water blockage generally occurs under a rapid increase of matric suction (Wildenschild et al. 2001). For a soil with local heterogeneity, the pores of different sizes are heterogeneously distributed. During a drying process, drainage usually occurs in larger pores first and then in smaller pores. If the matric suction is rapidly applied, pore water can be preferentially driven out of some interconnected large pores, forming islands with fully saturated small pores in the soil. As such, some amount of pore water is trapped in the islands by a closed band of unsaturated large pores. Hence, at the same applied matric suction, if the matric suction is applied rapidly, the amount of discharged water can be much less than expected, and the degree of saturation is unexpectedly large, as in the case of Test 3 (Figs. 3 and 5).

Fig. 4 shows that parameter τ increases with the applied matric suction. This is an expected result, since at lower saturation, the soil hydraulic conductivity becomes poorer so that it takes longer time for the soil to attain equilibrium. Fig. 4 also illustrates that the variations of C with the applied matric suction in Tests 1 and 2 are practically the same, whereas the variation of C in Test 3 is significantly different. The discrepancy can be attributed to the effect of water blockage in Test 3 during drying, which results in a locally heterogeneous moisture distribution in the unsaturated soil. In the case that the water blockage does not occur (Tests 1 and 2), coefficient C is a function of the applied matric suction only and independent of the changing rate of the matric suction, as implied by its very definition [Eq. (4)]. Hence, although the matric suction increases much faster in Test 2 than in Test 1, the calculated C values are quite similar for both tests, as shown in Fig. 4.

Figs. 6–8 present the experimental results and theoretical simulations of the nonequilibrium MSO experiments for the silt. Four different multistep outflow tests were performed to examine the effect of the magnitude and duration of the applied matric suction step on the outflow process. The schemes for applying matric suction steps in these experiments are depicted in Fig. 6, where the temporal variations of the instant degree of saturation are also given. Based on the temporal variations of the instant degree of saturation, parameters τ_i and C_i can be determined, from which the SWCC and HCF can be calculated. The calculated results are given in Figs. 7 and 8.

Fig. 6 shows that, at the same applied matric suction, the instant degree of saturation increases (or the outflow mass decreases) as the changing rate of matric suction increases. In Test 4, where the changing rate of matric suction is fastest, the instant degree of saturation is the largest (or the outflow mass is the smallest) at a specified matric suction. The stated observations can be explained by

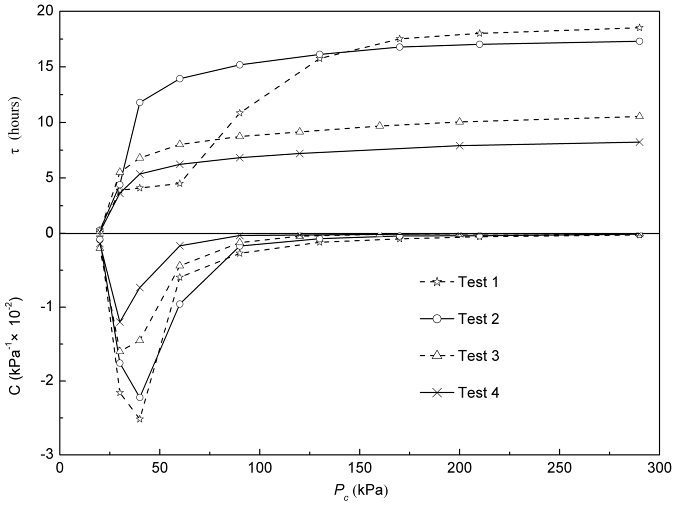

Fig. 7. Variations of parameters C and τ with applied matric suction for the silt

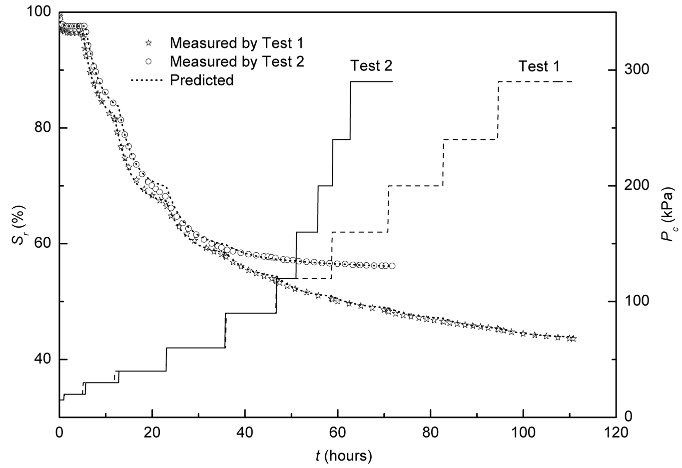

Fig. 9. Experimental schemes and results of the nonequilibrium MSO experiment on the silty clay

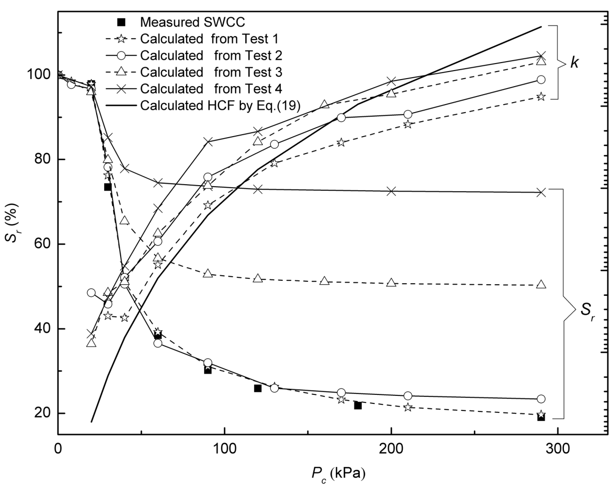

Fig. 8. Comparison of the calculated and measured hydraulic properties for the silt

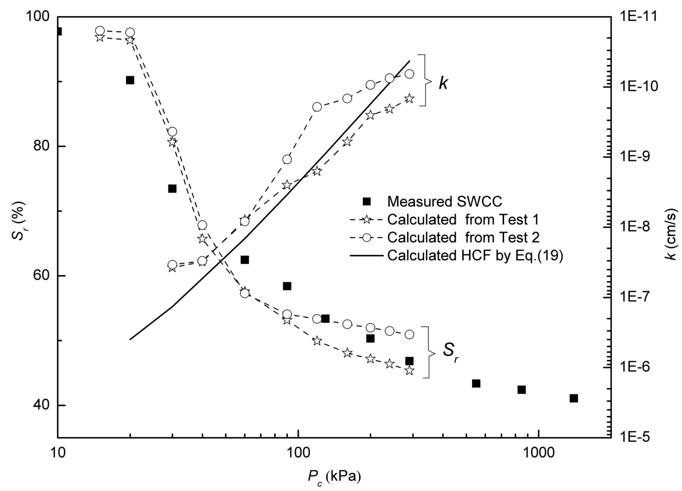

Fig. 10. Comparison of the calculated and measured hydraulic properties for the silty clay

invoking the water blockage mechanism that has been as discussed earlier. Clearly, the water blockage effect is quite pronounced in the tested silt. Comparison of the calculated variations of the instant saturation with the measurements demonstrates that the moisture redistributing process in the soil can be excellently described by Eq. (9), again implying that the moisture redistribution follows the exponential law.

Fig. 7 demonstrates that parameter τ increases with the applied matric suction and that water blockage effect exerts significant controls over the relationship between parameter C and the applied matric suction. In the case that there is no significant water blockage effect (Tests 1 and 2), parameter C is a function of the applied matric suction only and independent of the changing rate of suction. In Tests 3 and 4, the magnitudes and the durations of suction steps were generally larger than those of Tests 1 and 2, and a significant amount of pore water was blocked in the soil. This can also be clearly seen by comparing parameter C among different tests at matric suction of 40 kPa (Fig. 7), at which the magnitudes of C (or the water storage capacity) in Tests 3 and 4 become significantly lower (or larger) than those of Tests 1 and 2.

Fig. 8 illustrates that the calculated SWCCs based on Tests 1 and 2 agree very well with the one measured at equilibrium, whereas the calculated SWCCs based on Tests 3 and 4 deviate significantly from the measured SWCC. Accordingly, compared to Tests 3 and 4, the calculated HCFs from Tests 1 and 2 agree reasonably well with the estimation of Eqs. (19*a*) and (19*b*). The given results imply that the hydraulic properties as determined are not unique, and instead they depend upon the magnitude and the duration of each matric suction step applied in the experiment. Hence, in applying the proposed method, it is important to use a proper scheme to apply suction steps in the MSO experiment such that the proposed procedure takes less time than the conventional methods in determining the hydraulic properties, while it can still yield results with sufficient accuracy. As a general guide, some suggestions for applying the suction step in a MSO experiment will be given later.

Figs. 9 and 10 demonstrate the experimental results and theoretical simulations of the nonequilibrium MSO experiments on the silty clay. Two different schemes were adopted for applying the matric suction steps, denoted by Tests 1 and 2, respectively. It can be clearly seen from Fig. 9 that the calculated saturation agrees extremely well with the measurements, again implying that the

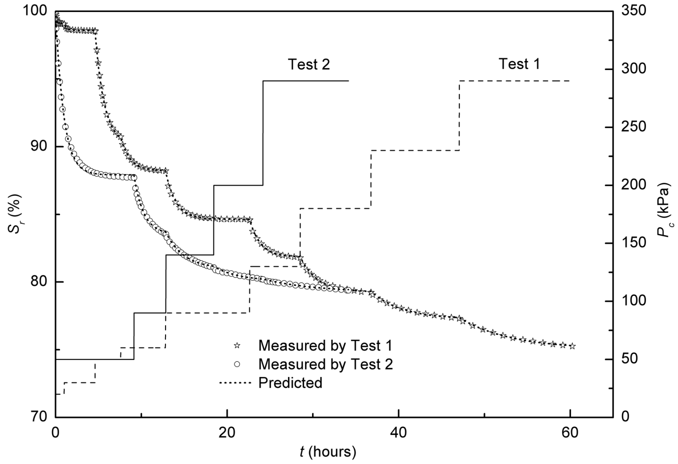

Fig. 11. Experimental schemes and results of the nonequilibrium MSO experiment on the clay

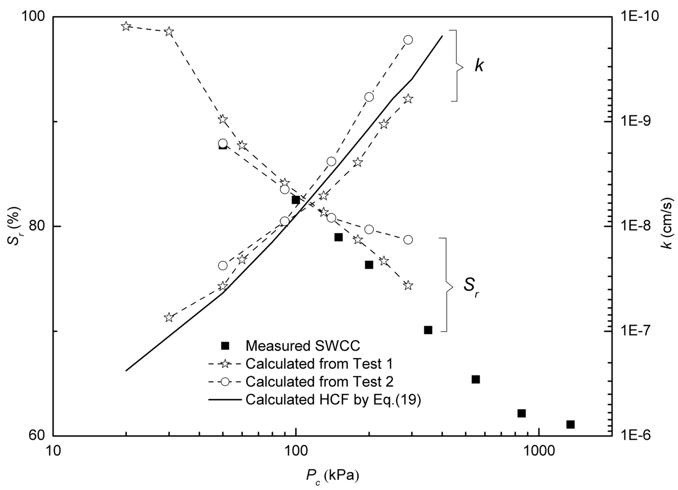

Fig. 12. Comparison of the calculated and measured hydraulic properties for the clay

temporal variation of the instant degree of saturation can be well addressed by using an exponential type of equation such as Eq. (9). The calculated SWCCs and HCFs based on the measured instant degree of saturation in the nonequilibrium MSO experiment are given in Fig. 10, where the measured SWCC are also given. For both tests, the calculated SWCCs agree reasonably well with the measurement over the whole range of matric suction in the tests, though the residual saturation is somewhat overestimated for Test 2. The discrepancy can be explained by invoking the mechanism of pore-water blockage in the soil during the drying process. Fig. 10 also shows that the calculated HCFs from both tests agree well with that inferred from Eqs. (19*a*) and (19*b*), though the one calculated from Test 1 yields a better result.

Figs. 11 and 12 present the experimental results and theoretical simulations for the clay. Two schemes were adopted to apply the matric suction steps, as shown in Fig. 11, where both the measured and calculated variations of the instant degree of saturation are given for comparison. Clearly, the calculations and the measurements are almost overlapped, implying that the temporal variation of saturation can be well addressed by using an exponential type of equation. The calculated and measured SWCCs are compared in Fig. 12. It can be seen that the SWCC calculated from Test 1 agrees well with the measurement up to about 300 kPa, i.e., the maximum applied suction. In contrast, however, the SWCC calculated from Test 2 deviates from the measurement when the matric suction becomes larger than 140 kPa, due to the water blockage effect. This effect can be clearly demonstrated in Fig. 11, where one can see that the instant degree of saturation quickly approaches to a constant when the matric suction increases above 140 kPa. Fig. 12 also shows that the calculated HCFs from both tests agree reasonably well with the HCF inferred from Eqs. (19*a*) and (19*b*).

Further Discussions

In a multistep outflow process, flow instability and water blockage may occur in the pores. In a poorly graded sandy soil, the flow instability can place important controls over the outflow process, if the applied matric suction is raised up to the air entry value. As a consequence, the outflow mass could be significantly different even when the applied suction is changed only slightly, resulting in significant errors in determining the hydraulic properties of unsaturated soils (as illustrated by Test 2 in Fig. 5). To minimize the adverse effects induced by flow instability, it is suggested here that small suction steps with a relatively long duration is adopted in the vicinity of air entry value.

The experimental results indicate that, if the size of the applied matric suction is large or the duration is short, the water blockage can occur, so that significant amount of pore water can be trapped in the small pores, resulting in the outflow mass much smaller (or the instant degree of saturation much larger) than the one from the test with a smaller suction step size and a longer duration (compare Tests 1 and 4 in Fig. 6). If the water blockage occurs, one can expect that significant errors can be induced in determining the hydraulic properties based on the proposed method (as illustrated by Tests 3 and 4 in Fig. 8).

Despite of the fact that flow instability and water blockage could occur sometimes, the nonequilibrium MSO experiment, with combination of the theory presented in section "Theory" can be effectively used to determine the hydraulic properties of unsaturated soils in a rapid way, if a proper scheme for applying suction steps was adopted in the MSO experiments (Test 1 in Figs. 5, 8, 10, and 12). As a general guideline, it is suggested here that (1) the air entry value of tested soil should be first estimated before the experiment, and then a suction step approximately equal to the air entry value be applied to the soil until the system attains equilibrium (to minimize the effect of flow instability); and (2) in the following suction steps, proper magnitudes and durations should be carefully chosen (to avoid the pore-water blockage). By the proper magnitudes of suction steps, those usually adopted in a conventional pressure plate experiment can be referred to. In addition, the duration of an applied suction step can be chosen in such a way that it must be as short as possible but still long enough to allow the temporal variation of outflow to appear clearly as an exponential-type curve.

Because a nonequilibrium MSO experiment can be finished in a period much shorter than its equilibrium counterpart (say the pressure plate measurement), a significant amount of time can be saved in determining the hydraulic properties of unsaturated soils by the proposed method. Fig. 13 compares the amounts of time required by the proposed method and those by the conventional pressure plate method (in the same suction range of 300 kPa) for the soils tested in this research. It is clearly shown that for all the tested soils the time amount required by the proposed method is several days only, which is much shorter than that by the pressure plate method. Interestingly, over the suction range (0–300 kPa) in the experiments, the time amounts required by the proposed method are quite

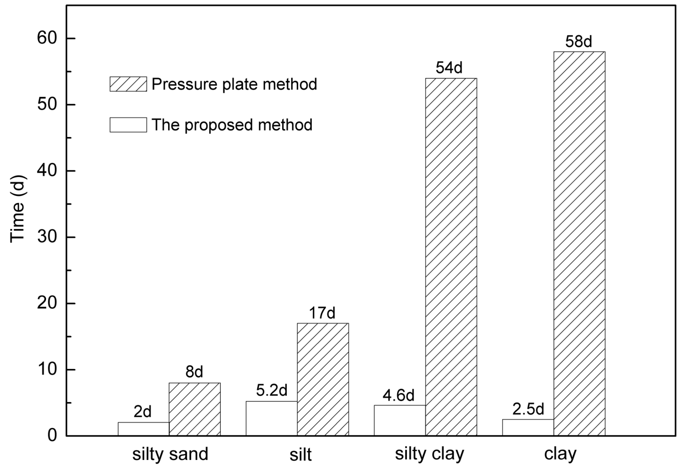

Fig. 13. Comparison of the measurement times by a pressure plate and the proposed method

comparable for all the tested soils. Particularly the required time amount for the tested clay is similar to that for the tested silty sand. The results imply that the proposed method is effective in quickly determining the hydraulic properties of unsaturated soils, especially for fine-grained soils.

Furthermore, if the theory developed in this study is correct, the reliability of the proposed methodology depends largely upon the accuracy of the measured outflow curve. To obtain reliable outflow measurements, the HAE ceramic disk must have a conductivity higher than the soil in the outflow experiment. Otherwise, the measured outflow curve would be polluted by the inferior conductivity of the ceramic stone. In the experiments, a one-bar and three-bar high air-entry value ceramic were used, and their saturated hydraulic conductivity were both approximately 10^{-7} cm/s (7.56×10^{-7} and 2.5×10^{-7} cm/s, respectively), which are generally higher than the range of the conductivity of the tested soils under unsaturated conditions. Nonetheless, the effect of the conductivity and thickness of the ceramic disk need to be examined in the future research.

In the proposed procedure, the volumetric deformation of sample is assumed negligible. Hence the procedure should apply only to the nonswelling and nonshrinking soils. For a swelling or collapsible soil, if the volumetric strain can be properly characterized during the variation of moisture, the proposed method might also be applicable, though its validity needs to be confirmed by future research. Due to the low pressure limit of the current TRIM system, the proposed method was applied to determine the hydraulic properties of sandy, silty, and clayey soils only in a relatively short range of matric suction (0–300 kPa). In addition, the proposed method yields an exponential expression for the temporal variation of saturation (or outflow mass). Although this expression has been shown to be applicable in the range of suction under consideration, its validity in a high-suction range has yet to be confirmed. All these problems could provide interesting topics for future research.

Summary and Conclusions

Based on the recent development of the continuum theory of porous media, a linear dynamic model is developed for describing the soil-water characteristics of unsaturated soils under transient conditions. The new model is used to characterize the evolution of the instant degree of saturation in the unsaturated soil during a multistep outflow experiment, in which the volumetric change of soils due to the variation of moisture is neglected. Within this context, a new procedure for rapidly determining the hydraulic properties of unsaturated soils is developed. A series of nonequilibrium multistep outflow experiments are performed on various types of unsaturated soils. The theoretical predictions are compared to the experimental results, showing the applicability of the proposed method.

It is shown that the evolution of the instant degree of saturation or the outflow mass generally follows an exponential law. Due to this feature, the effect of heredity can be effectively addressed in the evolution of saturation during a nonequilibrium multistep outflow experiment. Both experimental results and theoretical simulations illustrate that flow instability and water blockage can occur in the soil during nonequilibrium multistep outflow experiments. These effects can induce significant errors in determining the hydraulic properties of unsaturated soils based on the proposed procedure. The induced errors can be eliminated by introducing smaller matric suction steps with a relatively long duration in the MSO experiments. It is shown that if a proper scheme is adopted to apply the matric suction, the nonequilibrium MSO experiment, with combination of the proposed theory, can be effectively used to rapidly determine the hydraulic properties of unsaturated soils, including the soil-water characteristic relationship and the hydraulic conductivity function.

Appendix. Determination of Parameter t_i^{eq}

In the MSO experiment, equilibrium can be defined as such that the outflow water mass remains practically unchanged after a matric suction step is applied. Numerically, the equilibrium condition can be restated as follows: For step i, the equilibrium condition requires that the change of outflow water mass in a unit time (denoted by Δm^i) be not more than a specified small quantity ε, i.e., $\Delta m^i \leq \varepsilon$.

Accordingly, if the instant degree of saturation is introduced, the equilibrium condition requires that

$$\Delta S_r^i(t) = |S_r^i(t+\Delta t) - S_r^i(t)| \leq \delta \Delta t \tag{20}$$

where δ = small quantity; S_r^i = instant degree of saturation during step i; and ΔS_r^i = increment in the instant degree of saturation in a specified small time interval (Δt) within step i. Introducing Eq. (8), one obtains

$$\sum_{k=1}^{i} \left| \Delta p_c^k C_k \left[\exp\left(-\frac{t-t_k}{\tau_k}\right) - \exp\left(-\frac{t+\Delta t - t_k}{\tau_k}\right) \right] \right| \leq \delta \Delta t \tag{21}$$

The duration required for equilibrium, t_i^{eq}, is equal to the minimum value of $(t - t_i)$ at which Eq. (21) is satisfied. The Newton-Rhapson method can be used to solve Eq. (21). In calculation, $\delta = 1.0 \times 10^{-6}$ is assumed. It is remarkable that Eq. (9) includes the saturation changes inherited from all the previous nonequilibrium steps. Because the pore-water pressure dissipation process in the soil with a lower saturation is slower than that at a higher saturation, one can conveniently assume that $\Delta S_r^{i-1}(t) \ll \delta \Delta t$ at the $(i-1)$th matric suction step, so that at equilibrium

$$\left| \Delta p_c^i C_i \left[\exp\left(-\frac{t-t_i}{\tau_i}\right) - \exp\left(-\frac{t+\Delta t - t_i}{\tau_i}\right) \right] \right| \leq \delta \Delta t \tag{22}$$

From Eq. (22), one obtains

$$t_i^{eq} = (t - t_i)_{eq} \approx \tau_i \ln\left\{ \frac{\Delta p_c^i C_i [\exp(-\Delta t/\tau_i) - 1]}{\delta \Delta t} \right\} \tag{23}$$

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